Master formula for one-loop renormalization of bosonic SMEFT operators

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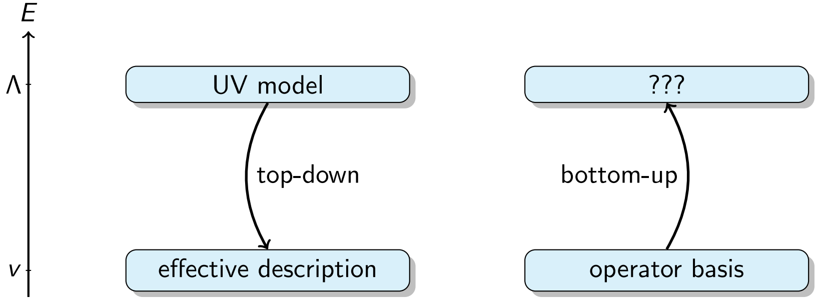
In collaboration with: Gerhard Buchalla, Alejandro Celis, and Jan-Niklas Toelstede
arXiv: 1904.07840

EFTs are an important tool in the search for new physics.



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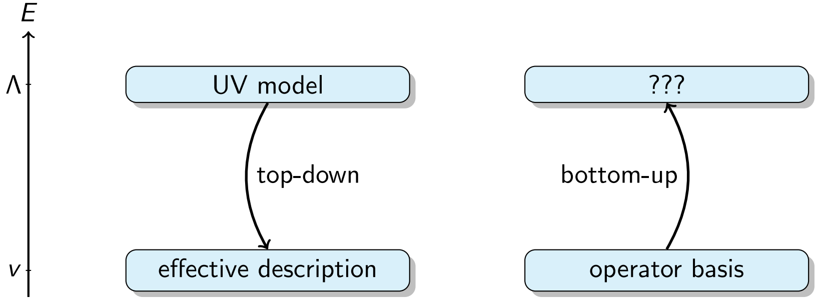

- We match to UV-models at scale Λ, but measure at scale v.
- The necessary RGEs also introduce an important operator mixing that was computed by Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014; JHEP].
- The RGEs are also implemented in `DSixTools`: [1704.04504,EPJC] and `Wilson`: [1804.05033, EPJC]

$\Rightarrow$ An independent cross check with a different approach would be very beneficial!

Master formula for one-loop renormalization of bosonic SMEFT operators

Part I: The SMEFT in our notation

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2}\mathcal{L}^{(6)} + \ldots$$

$$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$$

Part II: The Master Formula for 1-loop divergences [1710.06412,1904.07840]

Part III: The result [1904.07840]

$$\mathcal{Q}_\phi = \left(\phi^\dagger\phi\right)^3$$

$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda^2}\mathcal{L}^{(6)} + \ldots$ I: We have a mass gap to NP.

We assume there is a gap between the SM at v and the new physics (NP) at Λ.

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We work with the real representation of the Higgs doublet

$$H \equiv \frac{1}{\sqrt{2}}\left(\widetilde{\phi}, \phi\right) = i\tau^j \varphi_j,$$

with $\tau^0 = -\frac{i}{2}\mathbf{1}$ and $\tau^i = \frac{\sigma^i}{2}$. The covariant derivative then becomes

$$(D_\mu \varphi)_i = \partial_\mu \varphi_i + igW_\mu^a t_{Lij}^a \varphi_j + ig' B_\mu t_{Rij}^3 \varphi_j$$

t_L and t_R are generators of $SO(4)$ with the algebra

$$[t_L^a, t_L^b] = i\epsilon^{abc} t_L^c \qquad \{t_L^a, t_L^b\} = \frac{1}{2}\delta^{ab} \qquad \text{tr}\, t_L^a t_L^b = \delta^{ab}$$

$$[t_R^a, t_R^b] = i\epsilon^{abc} t_R^c \qquad \{t_R^a, t_R^b\} = \frac{1}{2}\delta^{ab} \qquad \text{tr}\, t_R^a t_R^b = \delta^{ab}$$

$$[t_L^a, t_R^b] = 0 \qquad \text{tr}\, t_L^a t_R^b = 0$$

$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \frac{1}{\Lambda^2}\mathcal{L}^{(6)} + \dots$ I: Our notation for the SM.

In this notation, we have

$$\begin{aligned}\mathcal{L}_{\mathsf{SM}} = &-\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\\ &+ \frac{1}{2}(D_\mu\varphi)_i(D^\mu\varphi)_i + \frac{m^2}{2}\varphi_i\varphi_i - \frac{\lambda}{8}(\varphi_i\varphi_i)^2\\ &+ \bar\psi i \not{D}\psi - \left(\bar\psi\sqrt{2}H\mathcal{Y}P_R\psi + \text{ h.c. }\right)\end{aligned}$$

with $\psi = (u, d, \nu, e)^T$ and $\mathcal{Y} = \mathrm{diag}\,(\mathcal{Y}_u, \mathcal{Y}_d, 0, \mathcal{Y}_e)$.

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \frac{1}{\Lambda^2}\mathcal{L}^{(6)} + \dots$$

I: Our notation for the SMEFT.

We use the operators defined in the Warsaw basis and focus on the bosonic ones first. Grzadkowski/Iskrzynski/Misiak/Rosiek [1008.4884,JHEP]

$\mathcal{Q}_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{Q}_\varphi$	$\frac{1}{8}(\varphi\varphi)^3$	$\mathcal{Q}_{\psi\varphi}$	$\frac{1}{\sqrt{2}}(\varphi\varphi)\bar{\psi}_L H \psi_R$
$\mathcal{Q}_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{Q}_{\varphi\Box}$	$\frac{1}{4}(\varphi\varphi)\Box(\varphi\varphi)$	$\mathcal{Q}_{\psi G}$	$\sqrt{2}\bar{\psi}\sigma^{\mu\nu} G_{\mu\nu}^A T^A H \psi$
$\mathcal{Q}_W$	$\epsilon_{ijk} W_\mu^{i\nu} W_\nu^{j\rho} W_\rho^{k\mu}$	$\mathcal{Q}_{\varphi D}$	$\frac{1}{4}(\varphi D_\mu \varphi)(\varphi D^\mu \varphi)$	$\mathcal{Q}_{\psi W}$	$\sqrt{2}\bar{\psi}\sigma^{\mu\nu} W_{\mu\nu}^i \sigma^i H \psi$
$\mathcal{Q}_{\widetilde{W}}$	$\epsilon_{ijk} \widetilde{W}_\mu^{i\nu} W_\nu^{j\rho} W_\rho^{k\mu}$		$-(\varphi t_R^3 D_\mu \varphi)(\varphi t_R^3 D^\mu \varphi)$	$\mathcal{Q}_{\psi B}$	$\sqrt{2}\bar{\psi}\sigma^{\mu\nu} B_{\mu\nu} H \psi$
$\mathcal{Q}_{\varphi X}$	$\frac{1}{2}(\varphi\varphi) X_{\mu\nu}^a X^{a\mu\nu}$	$\mathcal{Q}_{WB}$	$2\varphi t_L^i t_R^3 \varphi W_{\mu\nu}^i B^{\mu\nu}$	$\mathcal{Q}_{\varphi\psi}^{(1)}$	$(\bar{\psi}_L \gamma^\mu \psi_L)(\varphi i \overleftrightarrow{D}_\mu \varphi)$
$\mathcal{Q}_{\varphi\widetilde{X}}$	$\frac{1}{2}(\varphi\varphi) \widetilde{X}_{\mu\nu}^a X^{a\mu\nu}$			$\mathcal{Q}_{\varphi\psi}^{(3)}$	$(\bar{\psi}_L \sigma^i \gamma^\mu \psi_L)(\varphi i \overleftrightarrow{D}_\mu^i \varphi)$
		$\mathcal{Q}_{\widetilde{W}B}$	$2\varphi t_L^i t_R^3 \varphi \widetilde{W}_{\mu\nu}^i B^{\mu\nu}$	$\mathcal{Q}_{\varphi\psi}$	$(\bar{\psi}_R \gamma^\mu \psi_R)(\varphi i \overleftrightarrow{D}_\mu \varphi)$
	$X = G, W, B$			$\mathcal{Q}_{\varphi ud}$	$-(\varphi(it_R^1 + t_R^2) D_\mu \varphi)(\bar{u}_R \gamma^\mu d_R)$

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Consider the 15 bosonic operators first

$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda^2}\mathcal{L}^{(6)} + \dots$ I: The 1-loop SMEFT RGEs.

- n LO vertices $\rightarrow$ LO operators

- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO ops.

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$$-32\pi^2\epsilon\, \delta\mathcal{L}_{\text{div}} = \frac{C_j}{\Lambda^2}\left(K_{(i)}\delta_{ij} + K_{ij}\right)\mathcal{Q}_i$$

$$\beta_i \equiv 16\pi^2 \frac{dC_i}{d\ln\mu} = K_{ij}C_j$$

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- running of dim-4 altered by $\beta(c_{\text{SM}}) \sim \frac{m^2}{\Lambda^2}\,c_{\text{dim-6}}\,c_{\text{SM}}^n$
- running of dim-6 given by $\beta(c_{\text{dim-6}}) \sim c_{\text{dim-6}}\,c_{\text{SM}}^n$

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$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ We use the Background-Field Method...

starting from the generating functional:

$$Z[j,\rho,\bar\rho] = e^{iW[j,\rho,\bar\rho]} = \int [d\phi d\psi d\bar\psi] \quad e^{i(S[\phi,\psi,\bar\psi]+j\phi+\bar\psi\rho+\bar\rho\psi)},$$

$$\phi = \hat\phi + \phi_{qu}, \qquad\qquad \psi = \hat\psi + \psi_{qu},$$

$$\left(\frac{\delta S}{\delta \phi} + j\right)_{\phi=\hat\phi} = 0, \qquad \left(\frac{\delta S}{\delta \bar\psi} + \rho\right)_{\bar\psi=\hat{\bar\psi}} = 0, \qquad \left(\frac{\delta S}{\delta \psi} - \bar\rho\right)_{\psi=\hat\psi} = 0$$

$$\Rightarrow e^{iW_{L=1}} = \int [d\phi_{qu} d\psi_{qu} d\bar\psi_{qu}] \quad e^{iS^{(2)}[\hat\phi,\hat\psi,\hat{\bar\psi};\phi_{qu},\psi_{qu},\bar\psi_{qu}]}$$

Abbott '81

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ We use the Background-Field Method...

Quantum gauge fixing:

$$\mathcal{L}_{\text{g-f}} = -\frac{1}{2}\left[\left(\hat{D}^{\mu}_{G} G^{A}_{\mu}\right)^2 + \left(\partial_\mu B^\mu - ig'\varphi_i t^3_{Rij}\hat{\varphi}_j\right)^2 + \left(\hat{D}^{\mu}_{W} W^{a}_{\mu} - ig\varphi_i t^a_{Lij}\hat{\varphi}_j\right)^2\right]$$

Dittmaier/Grosse-Knetter [hep-ph/9505266]; Helset/Paraskevas/Trott [1803.08001,PRL]

- We work in the Feynman gauge.
- The terms proportional to φ will make the next steps easier.

Using the background covariant derivative

$$\hat{D}^{W}_{\mu} X = \partial_\mu X + ig[\hat{W}_\mu, X]$$

maintains background gauge invariance.

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ We use the Background-Field Method...

evaluating the one-loop functional

Neufeld/Gasser/Ecker hep-ph/9806436

$$e^{iW_{L=1}} = \int [d\phi d\psi d\bar{\psi}] \quad e^{iS^{(2)}[\hat{\phi},\hat{\psi},\hat{\bar{\psi}};\phi,\psi,\bar{\psi}]}$$

$$S^{(2)} = \frac{1}{2}\phi A\phi + \bar{\psi}B\psi + \phi\bar{\Gamma}\psi + \bar{\psi}\Gamma\phi$$

$$W_{L=1} = \frac{i}{2}\,\mathrm{Tr}\ln A - i\,\mathrm{Tr}\ln B - \frac{i}{2}\sum_{n=0}^{\infty}\frac{1}{n}\,\mathrm{Tr}\left(A^{-1}\bar{\Gamma}B^{-1}\Gamma - A^{-1}\Gamma^{T}B^{-1,T}\bar{\Gamma}^{T}\right)^{n}$$

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ . . . and Super-Heat-Kernel Expansion. . .

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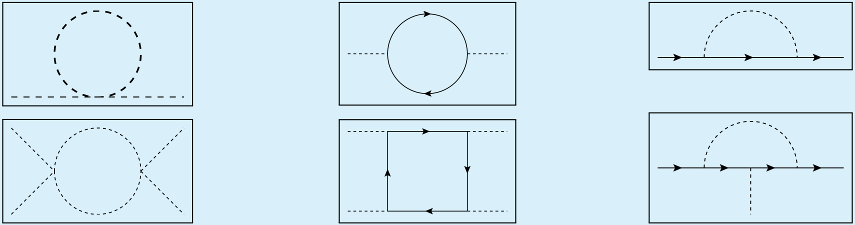

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ . . . and Super-Heat-Kernel Expansion. . .

Neufeld/Gasser/Ecker hep-ph/9806436

Introducing supermatrix algebra:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\begin{aligned} \text{sdet}\, M &= \det\left(A - BD^{-1}C\right)\det D^{-1} \\ \text{str}\, M &= \text{tr}\, A - \text{tr}\, D \\ \text{sdet}\, M &= e^{\text{str}\ln M} \end{aligned}$$

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The one-loop functional of $S^{(2)} = \frac{1}{2}\phi A\phi + \bar{\psi}B\psi + \phi\bar{\Gamma}\psi + \bar{\psi}\Gamma\phi$ becomes:

$$W_{L=1} = \frac{i}{2}\,\text{Str}\ln\Delta, \qquad \Delta = \begin{pmatrix} A & \bar{\Gamma} & -\Gamma^T \\ -\bar{\Gamma}^T & 0 & -B^T \\ \Gamma & B & 0 \end{pmatrix}$$

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ ... and Super-Heat-Kernel Expansion...

Applying the Heat-Kernel Expansion:

Donoghue/Golowich/Holstein '92

Neufeld/Gasser/Ecker hep-ph/9806436

$$W_{L=1} = \frac{i}{2}\,\mathrm{Str}\ln\Delta$$
$$= -\frac{i}{2}\int_0^\infty \frac{d\tau}{\tau}\int d^dx\,\mathrm{str}\,\langle x|e^{-\tau\Delta}|x\rangle$$

with the expansion in Seeley-DeWitt coefficients

$$\langle x|e^{-\tau\Delta}|x\rangle = \frac{i}{(4\pi)^{d/2}}\frac{e^{-\tau m^2}}{\tau^{d/2}}\sum_{n=0}^{\infty} a_n(x)\tau^n$$

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- The a_n can be computed, knowing the form of Δ.
- The UV-divergences of $W_{L=1}$ are the poles in $\frac{1}{\tau}$.

  $\Rightarrow$ only a_2 contributes!

$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2}$ . . . and Super-Heat-Kernel Expansion. . .

The Heat-Kernel Expansion extracts the $\frac{1}{\epsilon}$-poles of $W_{L=1}$.

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ ... and Super-Heat-Kernel Expansion...

The Heat-Kernel Expansion extracts the $\frac{1}{\epsilon}$-poles of $W_{L=1}$.

With Donoghue/Golowich/Holstein '92; Neufeld/Gasser/Ecker [hep-ph/9806436,PLB]

$$\Delta = (\partial_\mu + \Lambda_\mu)(\partial^\mu + \Lambda^\mu) + \Sigma$$

we get

$$W_{L=1,div} = \frac{1}{32\pi^2\epsilon}\int d^4x \,\mathrm{str}\left[\frac{1}{12}\Lambda_{\mu\nu}\Lambda^{\mu\nu} + \frac{1}{2}\Sigma\Sigma\right].$$

$$\Lambda_{\mu\nu} = \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu + [\Lambda_\mu, \Lambda_\nu]$$

- Specifying the Dirac structure of $S^{(2)}$, we can further evaluate the Dirac-traces.
- The resulting Master-Formula is purely algebraic (Matrix multiplication and traces). 'tHooft '73,NPB

$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2}$ … to find the Master Formula.

In the SM (and the EWChL), we have

$$\mathcal{L}_2^{\rm SM} = -\frac{1}{2}\phi^i A_i^j \phi_j + \bar\chi\left(i\partial\!\!\!/ - G\right)\chi + \bar\chi\Gamma^i\phi_i + \phi^i\bar\Gamma_i\chi,$$

with $A = (\partial^\mu + N^\mu)(\partial_\mu + N_\mu) + Y$ and $G \equiv (r + \rho_\mu\gamma^\mu)P_R + (l + \lambda_\mu\gamma^\mu)P_L$.

This gives

$$\mathcal{L}_{\rm div}^{\rm SM} = \frac{1}{32\pi^2\varepsilon}\left(\mathrm{tr}\left[\frac{1}{12}N^{\mu\nu}N_{\mu\nu} + \frac{1}{2}Y^2 - \frac{1}{3}\left(\lambda^{\mu\nu}\lambda_{\mu\nu} + \rho^{\mu\nu}\rho_{\mu\nu}\right)\right]\right.$$
$$\left. + \,\mathrm{tr}\left[2D^\mu l D_\mu r - 2lrlr\right] + \bar\Gamma\left(i\partial\!\!\!/ + iN\!\!\!/ + \frac{1}{2}\gamma^\mu G\gamma_\mu\right)\Gamma\right)$$

with

$$N_{\mu\nu} \equiv \partial_\mu N_\nu - \partial_\nu N_\mu + [N_\mu, N_\nu]\,,$$
$$\lambda_{\mu\nu} \equiv \partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu + i[\lambda_\mu,\lambda_\nu]\,, \qquad \rho_{\mu\nu} \equiv \partial_\mu\rho_\nu - \partial_\nu\rho_\mu + i[\rho_\mu,\rho_\nu]\,,$$
$$D_\mu l \equiv \partial_\mu l + i\rho_\mu l - il\lambda_\mu\,, \qquad D_\mu r \equiv \partial_\mu r + i\lambda_\mu r - ir\rho_\mu\,.$$

'tHooft '73,NPB; Buchalla/Catà/Celis/Knecht/CK [1710.06412,NPB]

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu}+\frac{1}{2}\Sigma^2$ ... to find the Master Formula.

In the bosonic SMEFT, we have

$$\mathcal{L}_2^{\text{SMEFT}} = \mathcal{L}_2^{\text{SM}} + \phi^i \left(a^{ij}_{\mu\nu} D^\mu D^\nu + 2 b^{ij}_\mu D^\mu + c^{ij} \right) \phi_j ,$$

with $a_{\mu\nu}, b_\mu, c \sim \frac{1}{\Lambda^2}$.

This gives

$$\mathcal{L}_{\text{div}}^{\text{SMEFT}} = \mathcal{L}_{\text{div}}^{\text{SM}} + \frac{1}{32\pi^2\varepsilon} \Bigg(\text{tr}\left[cY + \frac{1}{3} N_{\mu\nu}[D^\mu, b^\nu] + i\bar{\Gamma}\not{b}\Gamma - \frac{1}{6}\bar{\Gamma} i\overleftrightarrow{\not{D}}\!a\Gamma \right]$$
$$+ \text{tr}\left[\frac{1}{6} a^{\mu\nu} N_{\mu\lambda} N_\nu{}^\lambda - \frac{1}{24} a^\lambda_\lambda N_{\mu\nu} N^{\mu\nu} + \frac{1}{6} N_{\mu\lambda}[D_\nu, [D^\lambda, a^{\mu\nu}]] \right]$$
$$+ \text{tr}\left[\frac{1}{3} Y[D_\mu, [D_\nu, a^{\mu\nu}]] - \frac{1}{4} a^\lambda_\lambda Y^2 - \frac{1}{12} Y[D_\mu, [D^\mu, a^\lambda_\lambda]] \right] \Bigg).$$

Buchalla/Celis/CK/Toelstede [1904.07840]

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$\mathcal{Q}_\phi = \left(\phi^\dagger\phi\right)^3$ An explicit example

Starting from

$$\mathcal{Q}_\phi = \left(\phi^\dagger\phi\right)^3 = \frac{1}{8}\left(\varphi_i\varphi_i\right)^3\,,$$

we find

$$a_{ij}^{\mu\nu} = 0, \qquad b_{ij}^{\mu} = 0, \qquad c_{ij} = -\frac{3}{4}(\hat\varphi_a\hat\varphi_a)^2\delta_{ij} - 3(\hat\varphi_a\hat\varphi_a)\hat\varphi_i\hat\varphi_j,$$

$$Y_{ij} = \left(\left(\frac{\lambda}{2}+\frac{g^2}{4}\right)\hat\varphi_a\hat\varphi_a - m^2\right)\delta_{ij} + \left(\lambda - \frac{g^2}{4}\right)\hat\varphi_i\hat\varphi_j - g'^2(t_R^3\hat\varphi)_i(t_R^3\hat\varphi)_j.$$

$\mathcal{Q}_\phi = \left(\phi^\dagger\phi\right)^3$ An explicit example

Starting from

$$\mathcal{Q}_\phi = \left(\phi^\dagger\phi\right)^3 = \frac{1}{8}\left(\varphi_i\varphi_i\right)^3,$$

we find

$$a_{ij}^{\mu\nu} = 0, \qquad b_{ij}^{\mu} = 0, \qquad c_{ij} = -\frac{3}{4}(\hat\varphi_a\hat\varphi_a)^2\delta_{ij} - 3(\hat\varphi_a\hat\varphi_a)\hat\varphi_i\hat\varphi_j,$$

$$Y_{ij} = \left(\left(\frac{\lambda}{2} + \frac{g^2}{4}\right)\hat\varphi_a\hat\varphi_a - m^2\right)\delta_{ij} + \left(\lambda - \frac{g^2}{4}\right)\hat\varphi_i\hat\varphi_j - g'^2(t_R^3\hat\varphi)_i(t_R^3\hat\varphi)_j.$$

Therefore

$$\mathrm{tr}\, cY = -\left(54\lambda + \frac{9}{2}g^2 + \frac{3}{2}g'^2\right)\left(\phi^\dagger\phi\right)^3 + 24m^2\left(\phi^\dagger\phi\right)^2.$$

gives with $K_{(\phi)} = 6(3g^2 + g'^2 - \gamma_\phi)$

$$\beta_\phi \supseteq \left(54\lambda - \frac{27}{2}g^2 - \frac{9}{2}g'^2 + 6\gamma_\phi\right)C_\phi, \qquad \text{and} \qquad \beta_\lambda \supseteq 48\frac{m^2}{\Lambda^2}C_\phi.$$

$\mathcal{Q}_\phi = \left(\phi^\dagger \phi\right)^3$ Our results agree with the literature.

Buchalla/Celis/CK/Toelstede [1904.07840]

- We performed independent computations to cross check our results.
- We reproduce all RGE contributions of the 15 bosonic operators computed in
  Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014,JHEP]

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To compute the RGEs of the remaining operators, we have to extend our master formula by:

- Four-Fermion operators
- Fermionic tensor currents
- Mixed bosonic/fermionic derivatives

$\Rightarrow$ Stay tuned!

Master formula for one-loop renormalization of bosonic SMEFT operators — Summary —

- We derived a master formula for the $1/\epsilon$-poles based on the super-heat-kernel.

$$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$$

- It can be applied to a broad class of theories, like the SM, the bosonic sector of the SMEFT, or even the EWChL.
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- We reproduce the RGE contributions of the 15 bosonic operators that have been previously computed by Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014,JHEP]
- We are currently extending the master formula to include the full set of SMEFT operators.