Event Generation with Normalizing Flows: `i-flow`

— Particle Physics in Computing Frontier, IBS Daejeon —

Claudius Krause

Fermi National Accelerator Laboratory

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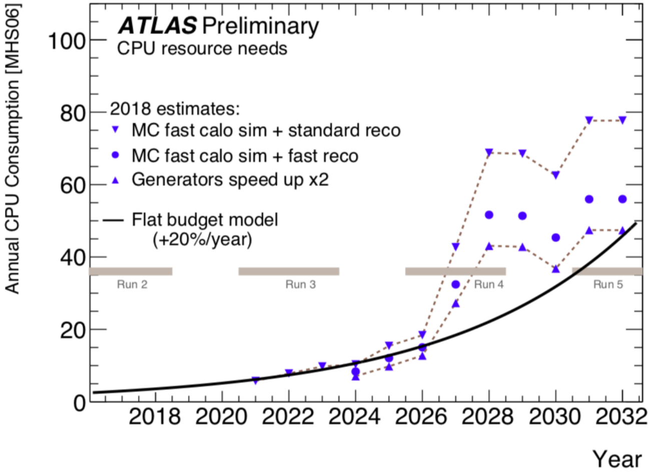
In collaboration with: Christina Gao, Stefan Höche, Joshua Isaacson, Holger Schulz
arXiv: 1912.abcde and arXiv: 1912.fghij

Monte Carlo Simulations are increasingly important.



https://twiki.cern.ch/twiki/bin/view/AtlasPublic/ComputingandSoftwarePublicResults

⇒ MC event generation is needed for signal and background predictions.

⇒ The required CPU time will increase in the next years.

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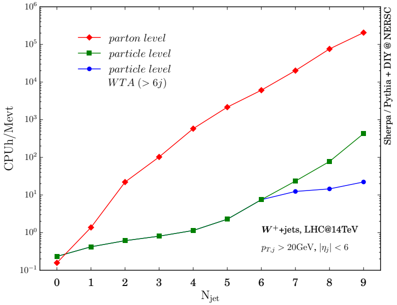

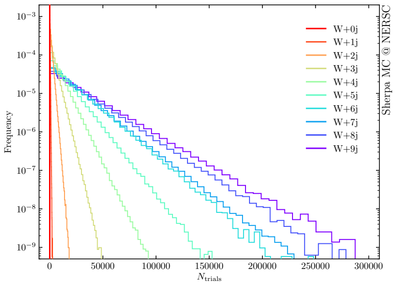

Stefan Höche, Stefan Prestel, Holger Schulz [1905.05120;PRD]

The bottlenecks for evaluating large final state multiplicities are

- a slow evaluation of the matrix element
- a low unweighting efficiency

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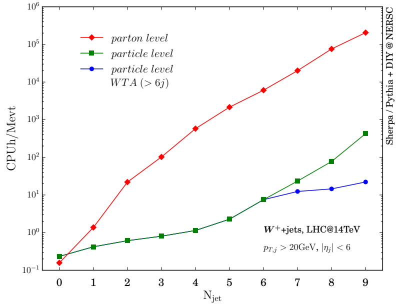

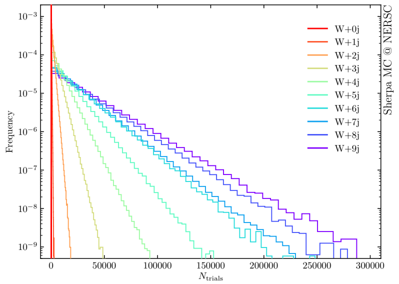

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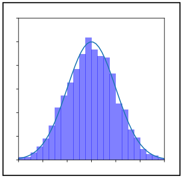

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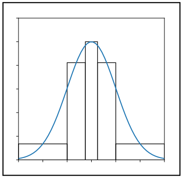
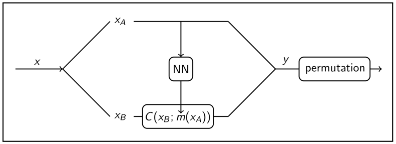
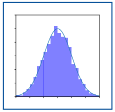


Part II: Existing Algorithms

Part III: Normalizing Flows

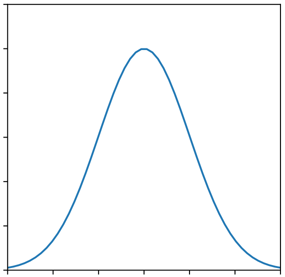


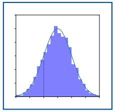


I: There are two problems to be solved...

$f(\vec{x})$

$d\sigma(p_i, \vartheta_i, \varphi_i)$





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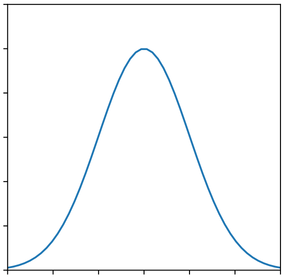
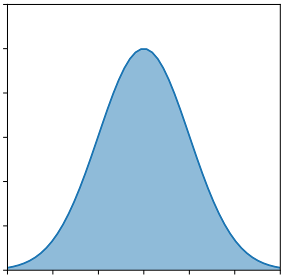
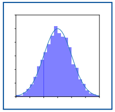
$$f(\vec{x}) \quad \Rightarrow \quad F = \int f(\vec{x})\, d^D x$$

$$d\sigma(p_i, \vartheta_i, \varphi_i) \quad \Rightarrow \quad \sigma = \int d\sigma(p_i, \vartheta_i, \varphi_i), \quad D = 3n_{\text{final}} - 4$$



$$\overset{?}{\Longrightarrow}$$





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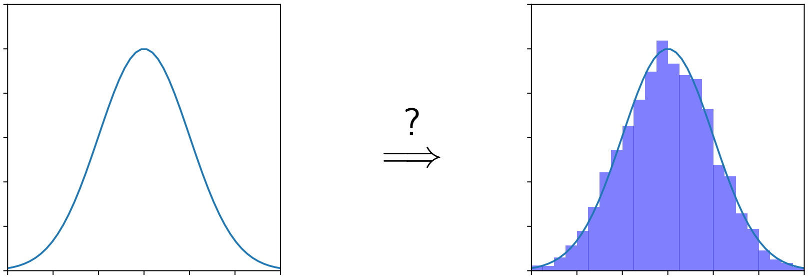
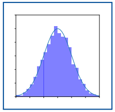
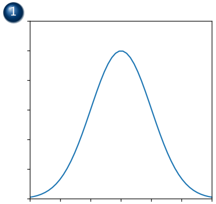
Given a distribution $f(\vec{x})$, how can we sample according to it?





I: ... but they are closely related.

1. Starting from a pdf, ...
2. ... we can integrate it and find its cdf, ...
3. ... to finally use its inverse to transform a uniform distribution.



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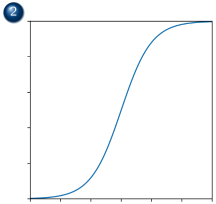

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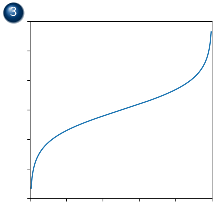

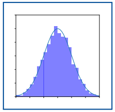

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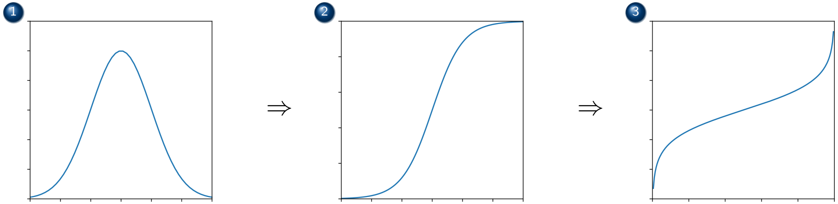
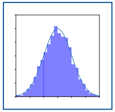


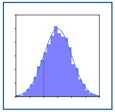
$\Rightarrow$ We need a fast and effective numerical integration!



I: Importance Sampling is very efficient for high-dimensional integration.

$$\int_0^1 f(x)\, dx \qquad \xrightarrow{\text{MC}} \qquad \frac{1}{N}\sum_i f(x_i) \qquad x_i \ldots \text{uniform}$$

$$= \int_0^1 \frac{f(x)}{q(x)}\, q(x) dx \qquad \xrightarrow[\text{importance sampling}]{\text{MC}} \qquad \frac{1}{N}\sum_i \frac{f(x_i)}{q(x_i)} \qquad x_i \ldots q(x)$$



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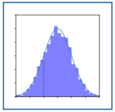
We therefore have to find a $q(x)$ that

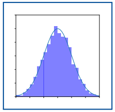
- approximates the shape of $f(x)$.
- is “easy” enough such that we can sample from its inverse cdf.



I: The unweighting efficiency measures the quality of the approximation $q(x)$.

- If $q(x) = \text{const.}$, each event x_i would require a weight of $f(x_i)$ to reproduce the distribution of $f(x)$. $\Rightarrow$ "Weighted Events"

- If $q(x) \propto f(x)$, all events would have the same weight as the distribution reproduces $f(x)$ directly. $\Rightarrow$ "Unweighted Events"



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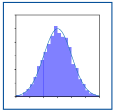
- To unweight, we need to accept/reject each event with probability $\frac{f(x_i)}{\max f(x)}$. The resulting set of kept events is unweighted and reproduces the shape of $f(x)$.
- The unweighting efficiency η gives the fraction of events that “survives” this procedure.

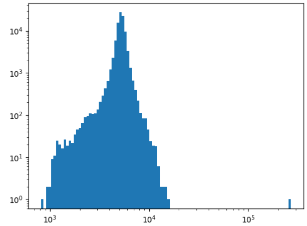
$$\eta = \frac{\#\ \text{accepted events}}{\#\ \text{all events}} = \frac{\text{mean } w}{\max w}, \text{ with } w_i = \frac{p(x_i)}{q(x_i)} = \frac{f(x_i)}{Fq(x_i)}.$$

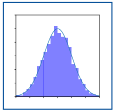


I: The usual definition of unweighting efficiency is unstable if many events are generated.

Problems of the old definition:

- The maximum grows with the number of events drawn.
- If more points are drawn than used in training, the chance for outliers increases a lot.
- Generating smaller subsets doesn't work, because we want a globally unweighted set of events.





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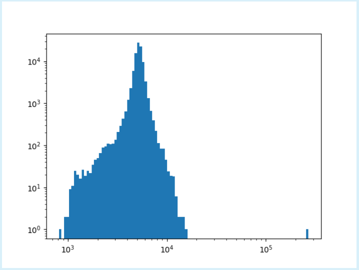
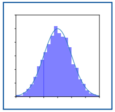


Our new definition:

- Assuming we used N_{opt} events during optimization, draw nN_{opt} events.
- Now, select m replicas of N_{opt} events each and find their maximum weight.
- Compute the total maximum as the median of the individual maxima.
- We expect a few overweight events that can either be discarded or included with their weights set to w_{max} (Requiring further control plots!).



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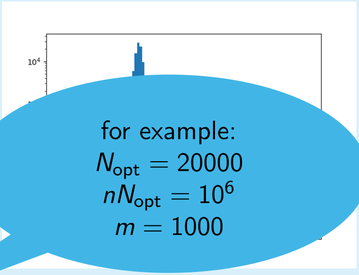

for example:
$N_{\text{opt}} = 20000$
$nN_{\text{opt}} = 10^6$
$m = 1000$

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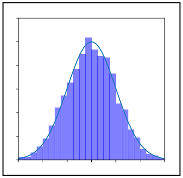
Event Generation with Normalizing Flows: `i-flow`



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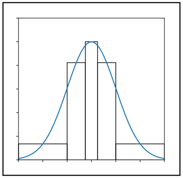

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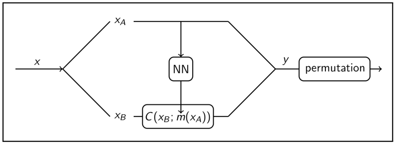
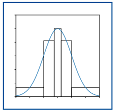
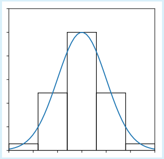




II: The VEGAS algorithm is very efficient.

The VEGAS algorithm Peter Lepage 1980

- assumes the integrand factorizes and bins the 1-dim projection.
- then adapts the bin edges such that area of each bin is the same.



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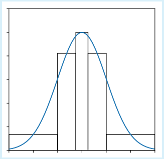

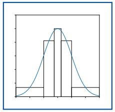

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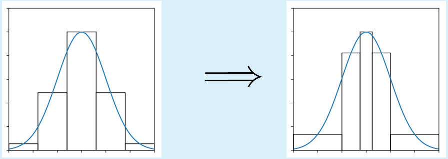
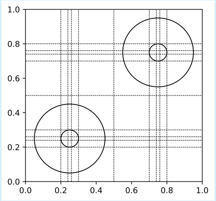


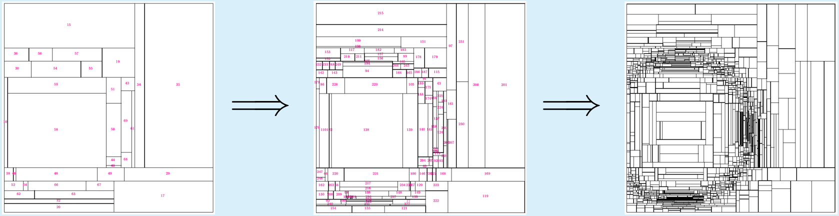
- It does have problems if the features are not aligned with the coordinate axes.
- The current python implementation also uses stratified sampling.



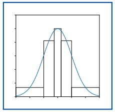
II: The `Foam` algorithm resolves correlations.

The `Foam` algorithm S. Jadach [physics/0203033]

- In the exploration phase, the integration domain is consecutively split into cells.
- In the generation phase, a cell is chosen at random and a point is drawn uniformly from within that cell.



illustrations from ICHEP 2002 slides, S. Jadach



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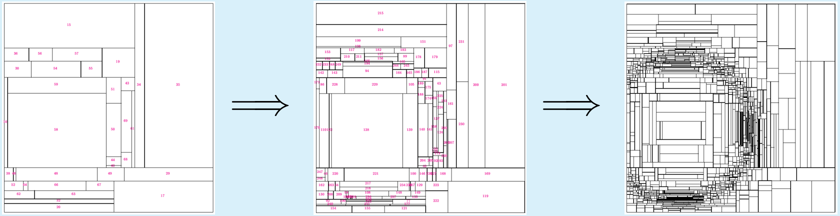

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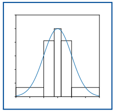
- It captures correlations.
- However, within each cell $q(x) = \text{const}$.



II: There are two different approaches to generate events with Machine Learning Techniques.

Generate events directly using GANs.

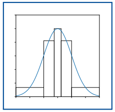
Suyong Choi et al. [next talk]; Bendavid [1707.00028]; Otten et al. [1901.00875]; Hashemi et al. [1901.05282]; Di Sipio et al. [1903.02433]; Butter et al. [1907.03764]; Carrazza et al. [1909.01359]

- ✓ Several orders of magnitude faster.
- ✓ Generates unweighted events directly.
- × Need existing event sample to train.
- × Results can be biased if not trained right.

Learn $q(x)$ to improve importance sampling.

Bendavid [1707.00028]; Klimek/Perelstein [1810.11509]; `i-flow` [this talk]

- ✓ Insufficient training just yields high uncertainties, no bias.
- ✓ Events are generated from scratch, no pre-existing set is needed.
- × Resulting set of events still needs to be unweighted.



II: The Loss function quantifies our goal.

We have different choices:

- Kullback-Leibler (KL) divergence:
  $D_{KL} = \int p(x) \log \frac{p(x)}{q(x)} dx \quad \approx \quad \frac{1}{N} \sum \frac{p(x_i)}{q(x_i)} \log \frac{p(x_i)}{q(x_i)}, \quad x_i \ldots q(x)$
- Pearson χ^2 divergence:
  $D_{\chi^2} = \int \frac{(p(x)-q(x))^2}{q(x)} dx \quad \approx \quad \frac{1}{N} \sum \frac{p(x_i)^2}{q(x_i)^2} - 1, \quad x_i \ldots q(x)$
- Exponential divergence:

$D_{exp} = \int p(x) \log \left(\frac{p(x)}{q(x)}\right)^2 dx \quad \approx \quad \frac{1}{N} \sum \frac{p(x_i)}{q(x_i)} \log \left(\frac{p(x_i)}{q(x_i)}\right)^2, \quad x_i \ldots q(x)$

We use the ADAM optimizer for stochastic gradient descent:

- The learning rate for each parameter is adapted separately, but based on previous iterations.
- This is effective for sparse and noisy functions.

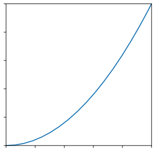
Kingma/Ba [arXiv:1412.6980]

II: Using the NN as coordinate transform is too costly.

We could use the NN as nonlinear coordinate transform:

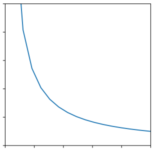
- We use a deep NN with n_{dim} nodes in the first and last layer to map a uniformly distributed x to a target $q(x)$.
- The distribution induced by the map $y(x)$ (=NN) is given by the Jacobian of the map:
  $q(y) = q(y(x)) = \left|\frac{\partial y}{\partial x}\right|^{-1}$

Klimek/Perelstein [arXiv:1810.11509]

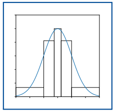


$y = x^2$

$\xrightarrow{\text{Jacobian}}$



$\left|\frac{\partial y}{\partial x}\right|^{-1} = \frac{1}{2x}$

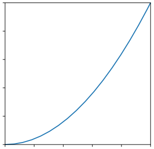


II: Using the NN as coordinate transform is too costly.

We could use the NN as nonlinear coordinate transform:

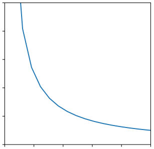
- We use a deep NN with n_{dim} nodes in the first and last layer to map a uniformly distributed x to a target $q(x)$.
- The distribution induced by the map $y(x)$ (=NN) is given by the Jacobian of the map:
  $q(y) = q(y(x)) = \left|\frac{\partial y}{\partial x}\right|^{-1}$

Klimek/Perelstein [arXiv:1810.11509]



$y = x^2$

$\xrightarrow{\text{Jacobian}}$

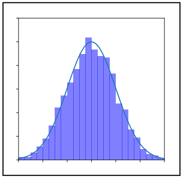


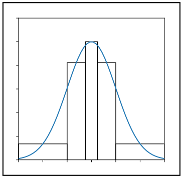
$\left|\frac{\partial y}{\partial x}\right|^{-1} = \frac{1}{2x}$

$\Rightarrow$ The Jacobian is needed to evaluate the loss and to sample. However, it scales as $\mathcal{O}(n^3)$ and is too costly for high-dimensional integrands!

Event Generation with Normalizing Flows: `i-flow`

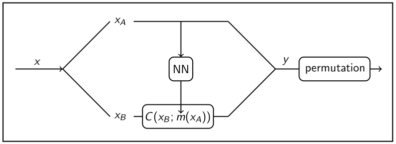
Part I: Monte Carlo Integration and Importance Sampling

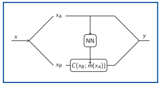




Part II: Existing Algorithms

Part III: Normalizing Flows



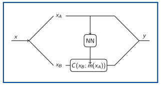


III: Normalizing Flows are numerically cheaper.

A Normalizing Flow:

- is a bijective, smooth mapping between two statistical distributions.
- is composed of a series of easy transformations, the *"Coupling Layers"*.
- is still flexible enough to learn complicated distributions.

⇒ The NN does not learn the transformation, but the parameters of a series of easy transformations.



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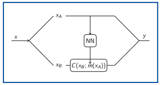
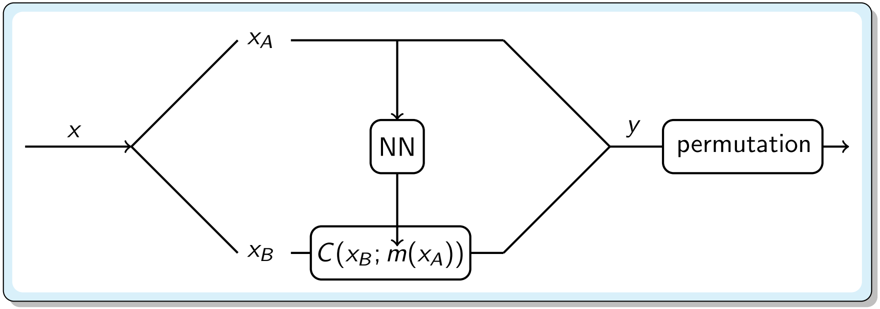
$\Rightarrow$ The NN does not learn the transformation, but the parameters of a series of easy transformations.

- The idea was introduced as "Nonlinear Independent Component Estimation" (NICE) in Dinh et al. [arXiv:1410.8516].
- In Rezende/Mohamed [arXiv:1505.05770], Normalizing Flows were first discussed with planar and radial flows.
- Our approach follows the ideas of Müller et al. [arXiv:1808.03856], but with the modifications of Durkan et al. [arXiv:1906.04032].
- Our code uses `TensorFlow 2.0`, `www.tensorflow.org`.



III: The Coupling Layer is the fundamental Building Block



forward:

$$y_A = x_A$$
$$y_{B,i} = C(x_{B,i}; m(x_A))$$

inverse:

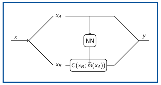
$$x_A = y_A$$
$$x_{B,i} = C^{-1}(y_{B,i}; m(x_A))$$

The C are numerically cheap, invertible, and separable in $x_{B,i}$.

Jacobian:

$$\left|\frac{\partial y}{\partial x}\right| = \begin{vmatrix} 1 & \frac{\partial C}{\partial x_A} \\ 0 & \frac{\partial C}{\partial x_B} \end{vmatrix} = \Pi_i \frac{\partial C(x_{B,i}; m(x_A))}{\partial x_{B,i}}$$

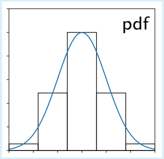
$$\Rightarrow \mathcal{O}(n)$$

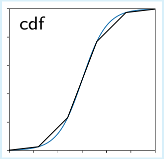


III: The Coupling Function is a piecewise approximation to the cdf.

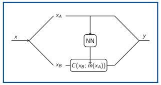
piecewise linear coupling function:

Müller et al. [arXiv:1808.03856]





The NN predicts the pdf bin heights Q_i.



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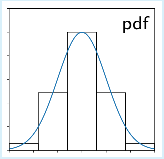
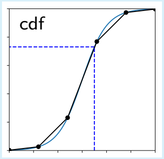


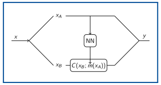


$$C = \sum_{k=1}^{b-1} Q_k + \alpha Q_b$$

$$\alpha = \frac{x-(b-1)w}{w}$$

$$\left| \frac{\partial C}{\partial x_B} \right| = \Pi_i \frac{Q_{b_i}}{w}$$

The NN predicts the pdf bin heights Q_i.



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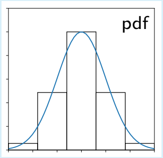

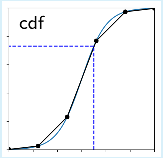

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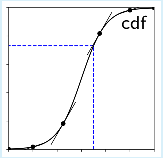
The NN predicts the pdf bin heights Q_i.

rational quadratic spline coupling function:

Durkan et al. [arXiv:1906.04032]

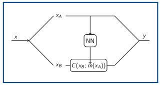
Gregory/Delbourgo [IMA Journal of Numerical Analysis, '82]



$$C = \frac{a_2\alpha^2 + a_1\alpha + a_0}{b_2\alpha^2 + b_1\alpha + b_0}$$

- still rather easy
- more flexible

The NN predicts the cdf bin widths, heights, and derivatives that go in $a_i \& b_i$.



III: We need $\mathcal{O}(\log n)$ Coupling Layers.

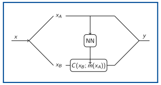
How many Coupling Layers do we need?

- Enough to learn all correlations between the variables.
- As few as possible to have a fast code.

- This depends on the applied permutations and the $x_A - x_B$-splitting:
  (pppttt)$\leftrightarrow$(tttppp) *vs.* (ppppttt)$\leftrightarrow$(ppttpp)$\leftrightarrow$(ttpppp)

- More pass-through dimensions (p) means more points required for accurate loss.
- Fewer pass-through dimensions means more CLs needed.

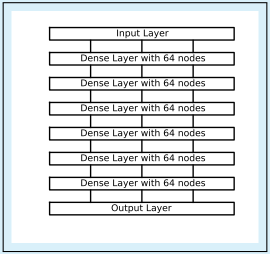
- For $\#p \approx \#t$, we can prove: $\boxed{4 \leq \#CLs \leq 2\log_2 n_{dim}}$

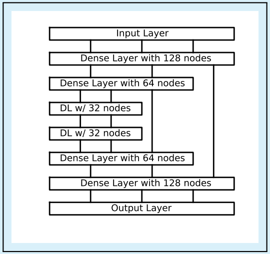


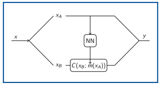
III: We utilize different NN architectures.

Available Architectures: Müller et al. [arXiv:1808.03856]

"Fully Connected" Neural Net (NN): "U-shaped" Neural Net (Unet):


Input Layer
Dense Layer with 64 nodes
Dense Layer with 64 nodes
Dense Layer with 64 nodes
Dense Layer with 64 nodes
Dense Layer with 64 nodes
Dense Layer with 64 nodes
Output Layer


Input Layer
Dense Layer with 128 nodes
Dense Layer with 64 nodes
DL w/ 32 nodes
DL w/ 32 nodes
Dense Layer with 64 nodes
Dense Layer with 128 nodes
Output Layer



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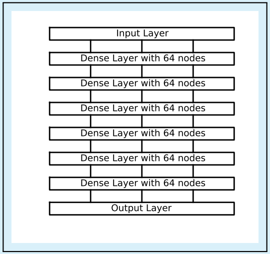
Input Layer
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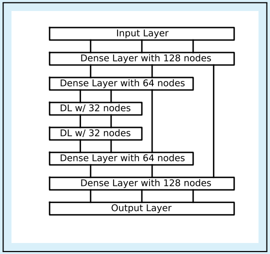
Dense Layer with 64 nodes
Dense Layer with 64 nodes
Dense Layer with 64 nodes
Dense Layer with 64 nodes
Dense Layer with 64 nodes
Output Layer

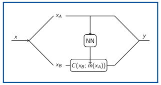
"U-shaped" Neural Net (Unet):


Input Layer
Dense Layer with 128 nodes
Dense Layer with 64 nodes
DL w/ 32 nodes
DL w/ 32 nodes
Dense Layer with 64 nodes
Dense Layer with 128 nodes
Output Layer

There are different ways to encode the input dimensions x_A.
For example $x_A = (0.2, 0.7)$:

- direct: $x_i = (0.2, 0.7)$
- one-hot (8 bins): $x_i = ((0,1,0,0,0,0,0,0),(0,0,0,0,0,1,0,0))$
- one-blob (8 bins): $x_i = ((0.55, 0.99, 0.67, 0.16, 0.01, 0, 0, 0),$
  $(0, 0, 0.01, 0.11, 0.55, 0.99, 0.67, 0.16))$

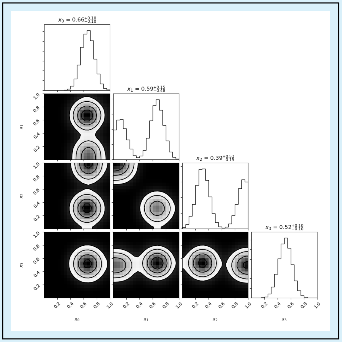
Müller et al. [arXiv:1808.03856]



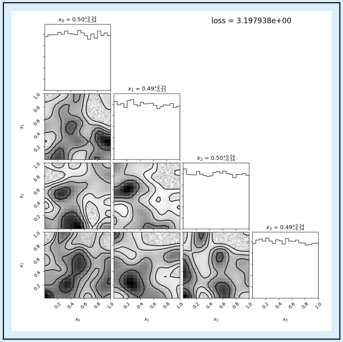
III: The 4-d Camel function illustrates the learning of `i-flow`.

Our test function: 2 Gaussian peaks, randomly placed in a 4d space.

Target Distribution:



Before training:



- Final Integral: 0.0063339(41)
- `VEGAS` plain: 0.0063349(92)
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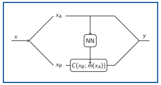
- Trained efficiency: 14.8 % Untrained efficiency: 0.6 %



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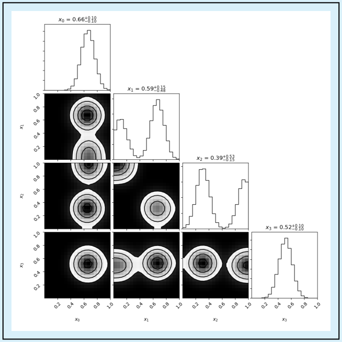
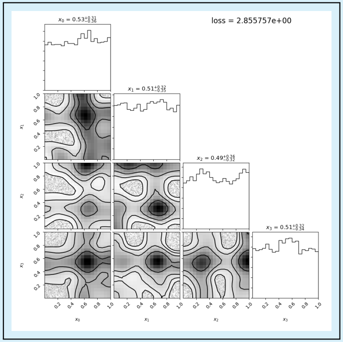


After 5 epochs:



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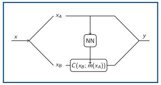

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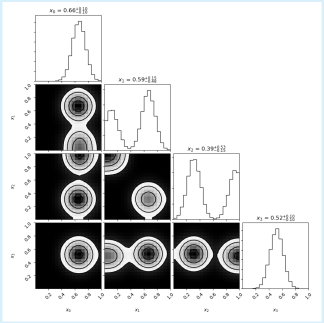
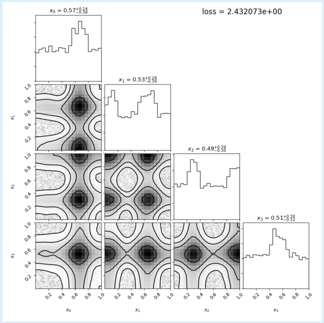


After 10 epochs:



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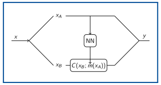

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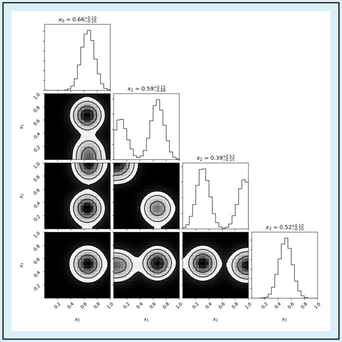
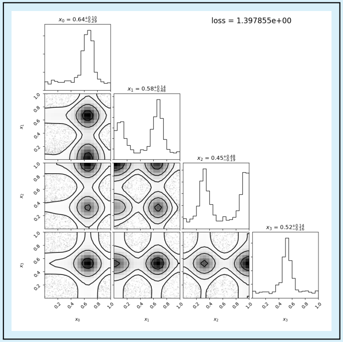


After 25 epochs:



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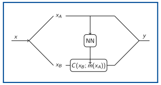

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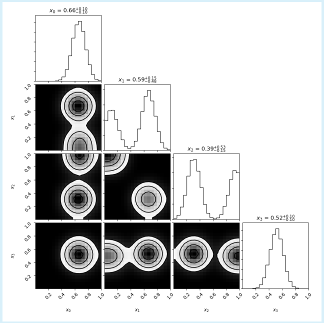
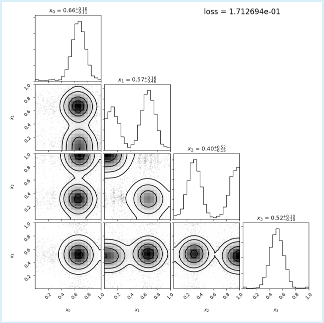


After 100 epochs:



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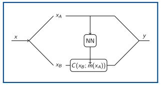

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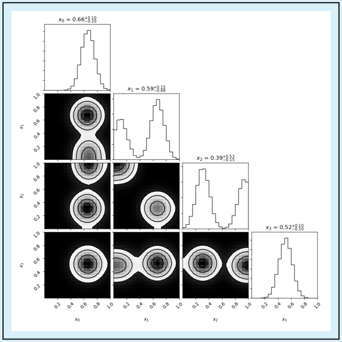
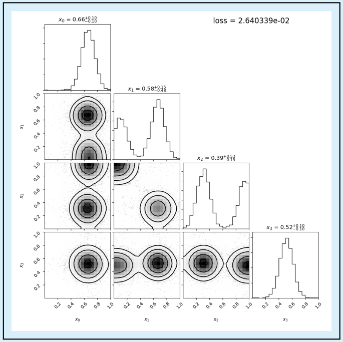


After 200 epochs:



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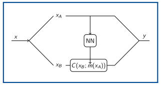
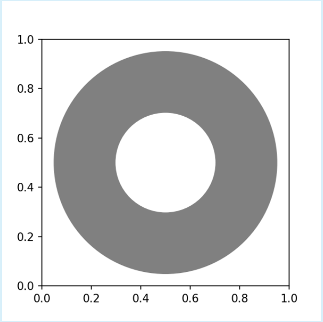


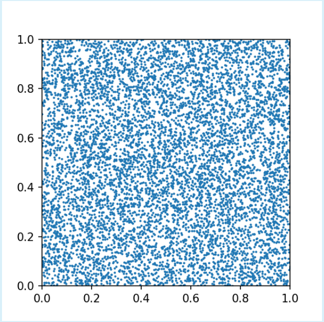
III: `i-flow` also learns hard, non-trivial cuts.

Our test function: a $2d$ ring function.

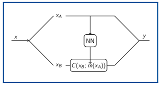
Target Distribution:



Before training:



- Final cut efficiency: 89 % Untrained efficiency: 51 %
- Integral: 0.510508 Estimated integral: 0.5112 ± 0.0006



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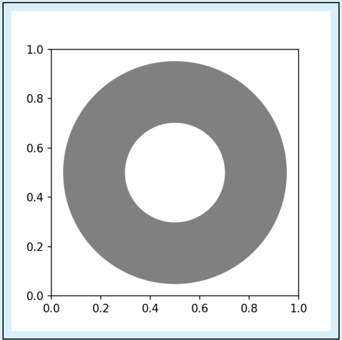
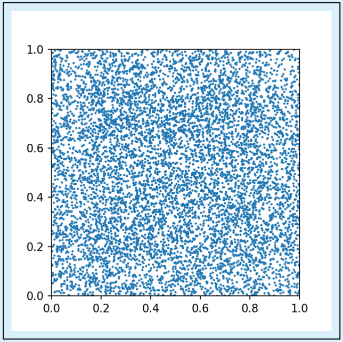
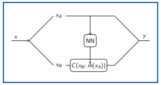


After 10 epochs:



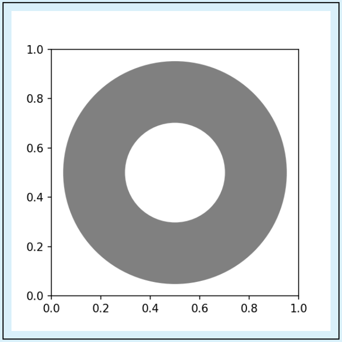
- Final cut efficiency: 89 % Untrained efficiency: 51 %
- Integral: 0.510508 Estimated integral: 0.5112 ± 0.0006



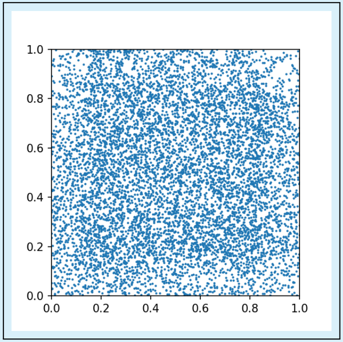
III: `i-flow` also learns hard, non-trivial cuts.

Our test function: a $2d$ ring function.

Target Distribution:



After 20 epochs:



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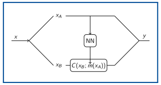

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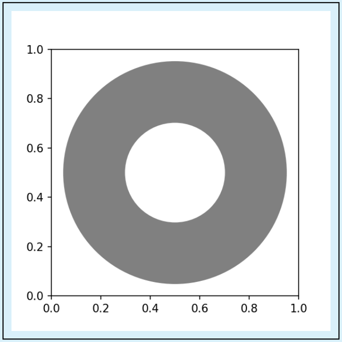
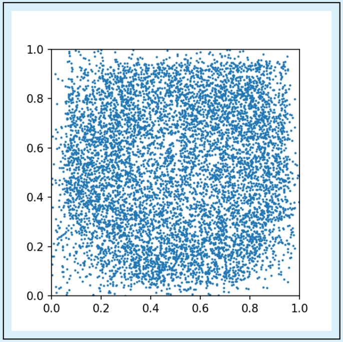


After 50 epochs:



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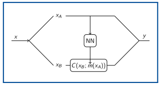

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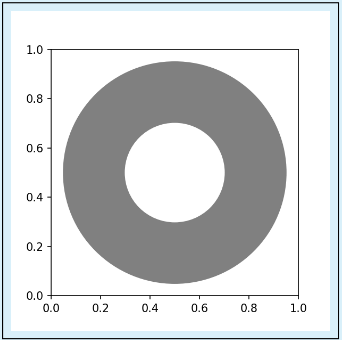
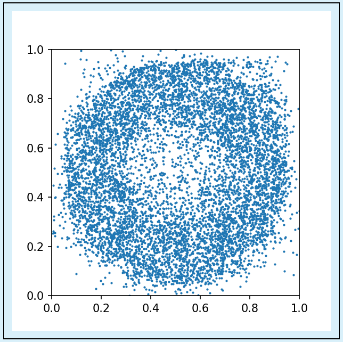


After 100 epochs:



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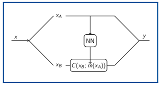

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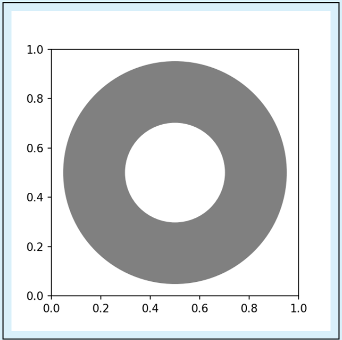
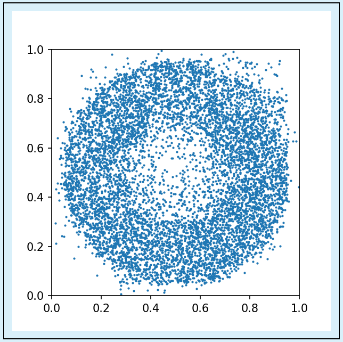


After 200 epochs:



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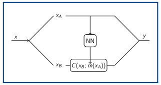

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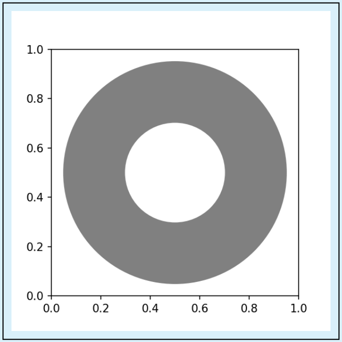
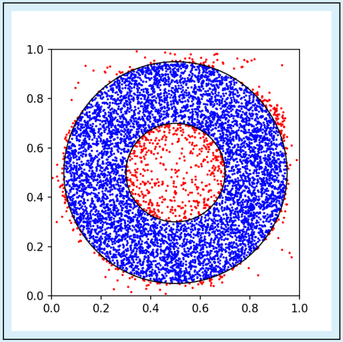


Final Distribution (500 epochs):



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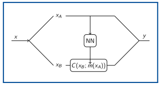


III: Sherpa needs a high-dimensional integrator.

Sherpa is a Monte Carlo event generator for the **S**imulation of **H**igh-**E**nergy **R**eactions of **PA**rticles. We use Sherpa to

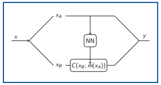
- compute the matrix element of the process.
- map the unit-hypercube of our integration domain to momenta and angles. To improve efficiency, Sherpa uses a recursive multichannel algorithm.

$$\boxed{\Rightarrow n_{dim} = \underbrace{3n_{final} - 4}_{\text{kinematics}} + \underbrace{n_{final} - 1}_{\text{multichannel}}}$$

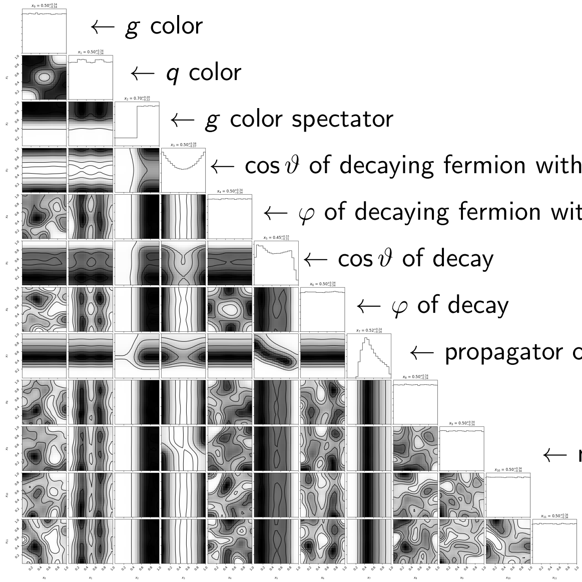
- However, the `COMIX++` ME-generator uses color-sampling, so we should also integrate over final state color configurations. While this improves the efficiency, it is not possible to handle group processes like $W + nj$ with a single flow.

$$\boxed{\Rightarrow n_{dim} = 4n_{final} - 4 + 2n_{color}}$$

https://sherpa.hepforge.org/



III: An easy example: $e^+e^- \to 3j$.



$\leftarrow$ g color

$\leftarrow$ q color

$\leftarrow$ g color spectator

$\leftarrow$ $\cos\vartheta$ of decaying fermion with beam

$\leftarrow$ φ of decaying fermion with beam

$\leftarrow$ $\cos\vartheta$ of decay

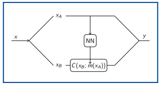
$\leftarrow$ φ of decay

$\leftarrow$ propagator of decaying fermion

$\leftarrow$ multichannel

Target distribution

with learning color



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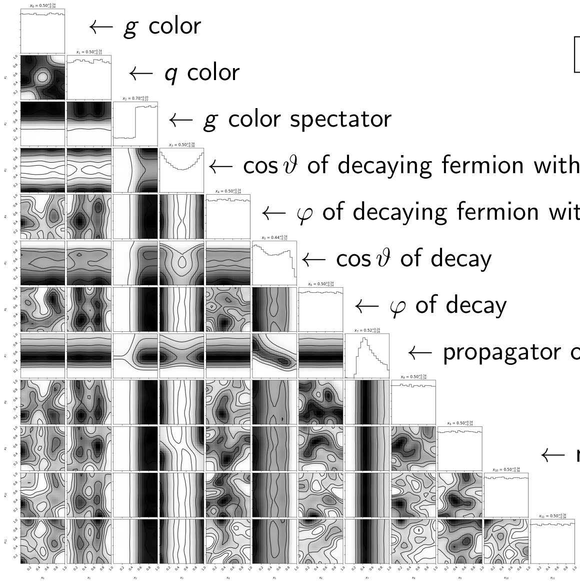
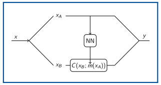


Learned distribution

with learning color



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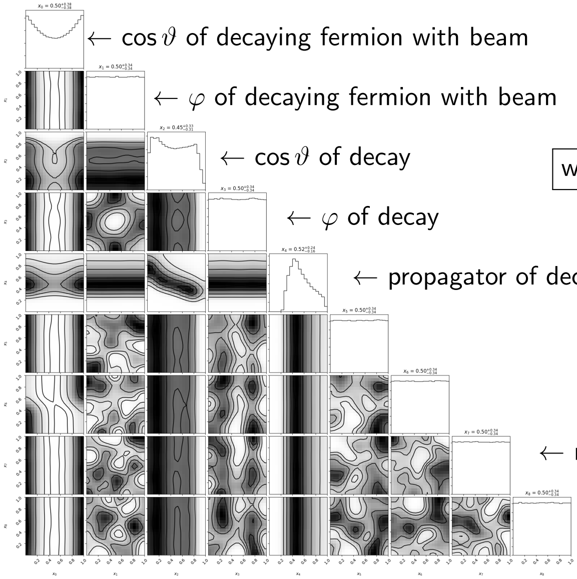


$\leftarrow$ $\cos\vartheta$ of decaying fermion with beam

$\leftarrow$ φ of decaying fermion with beam

$\leftarrow$ $\cos\vartheta$ of decay

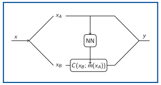
$\leftarrow$ φ of decay

$\leftarrow$ propagator of decaying fermion

$\leftarrow$ multichannel

$\boxed{\text{Target distribution}}$

$\boxed{\text{without learning color}}$



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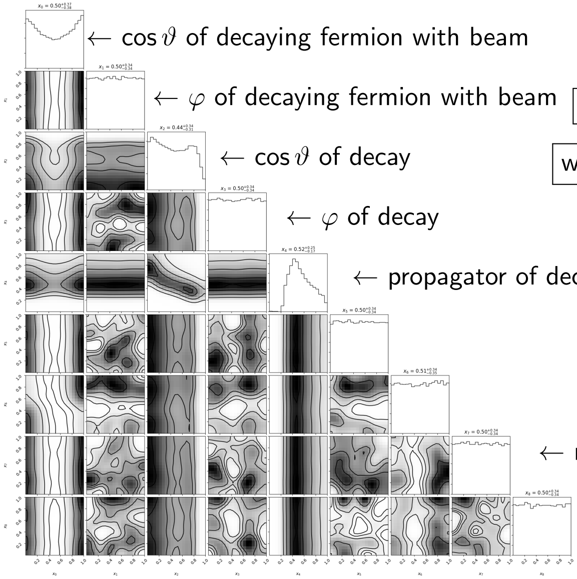

$\leftarrow$ $\cos\vartheta$ of decaying fermion with beam

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Learned distribution

without learning color

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$\leftarrow$ propagator of decaying fermion

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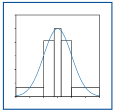
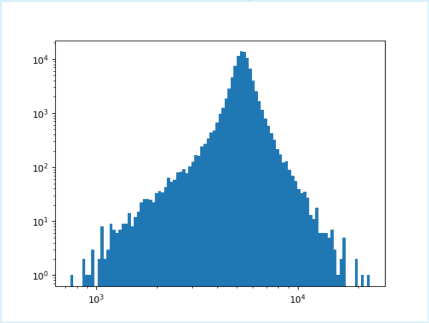


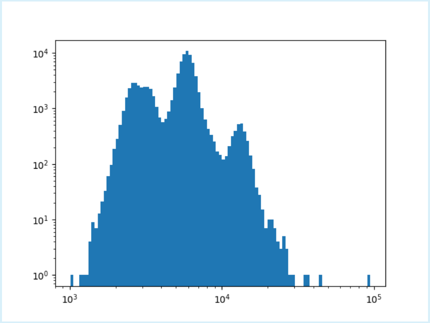
III: Comparing $e^+e^- \to 3j$ with and without learning color.

with learning color

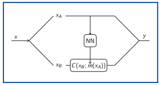


- $\sigma = 4879.8 \pm 5.3$pb
- $\eta_{\text{new}} = 45\%$
- Cut efficiency: 92 %
- 20 overweight events in 100k

without learning color



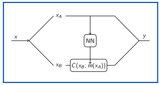
- $\sigma = 4883.5 \pm 8.5$pb
- $\eta_{\text{new}} = 25\%$
- Cut efficiency: 92 %
- 20 overweight events in 100k



III: Hyperparameter Optimization is difficult.

There are many hyperparameters that we have to optimize:

- network architecture:
  DNN vs. U-Net, # layers, # nodes
- learning schedule:
  schedule function (const., exponential, . . .), initial learning rate, decay rate and step size, . . .
- training:
  which loss function, # epochs, # samples per epoch
- normalizing flow specific:
  # (input/output) bins, input encoding (one blob, direct), how to split dims inside CL, # CLs, which function in the CLs



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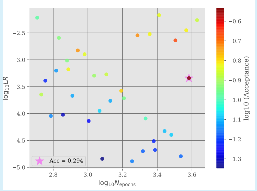
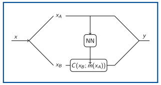
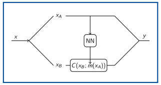
- We scan a subset of the hyperparameters with a Sobol sequence on NERSC.
- We focus on the process $pp \to W + 1j$.
- We apply the best point to $pp \to W + nj$.





III: High Multiplicities are still difficult to learn.

unweighting efficiency $\langle w\rangle/w_{\max}$		LO QCD $n=0$	$n=1$	$n=2$	$n=3$
$W^+ + n$ jets	Sherpa 2.2.7	$2.4\cdot10^{-1}$	$3.2\cdot10^{-2}$	$7.6\cdot10^{-3}$	$1.8\cdot10^{-3}$
	Sherpa 3.x.y	$2.5\cdot10^{-1}$	$2.9\cdot10^{-2}$	$7.3\cdot10^{-3}$	$2.0\cdot10^{-3}$
	NN+NF	$5.8\cdot10^{-1}$	$1.3\cdot10^{-1}$	$1.2\cdot10^{-2}$	$2.1\cdot10^{-3}$
	Gain	**2.3**	**4.3**	**1.7**	**1.1**
$W^- + n$ jets	Sherpa 2.2.7	$3.1\cdot10^{-1}$	$2.8\cdot10^{-2}$	$7.9\cdot10^{-3}$	$2.5\cdot10^{-3}$
	Sherpa 3.x.y	$2.4\cdot10^{-1}$	$4.4\cdot10^{-2}$	$9.4\cdot10^{-3}$	$2.1\cdot10^{-3}$
	NN+NF	$6.0\cdot10^{-1}$	$1.2\cdot10^{-1}$	$1.7\cdot10^{-2}$	$2.4\cdot10^{-3}$
	Gain	**2.5**	**2.6**	**1.8**	**1.1**
$Z + n$ jets	Sherpa 2.2.7	$2.9\cdot10^{-1}$	$3.6\cdot10^{-2}$	$1.4\cdot10^{-2}$	$3.1\cdot10^{-3}$
	Sherpa 3.x.y	$4.3\cdot10^{-1}$	$3.9\cdot10^{-2}$	$1.4\cdot10^{-2}$	$3.3\cdot10^{-3}$
	NN+NF	$5.1\cdot10^{-1}$	$1.3\cdot10^{-1}$	$1.9\cdot10^{-2}$	$3.1\cdot10^{-3}$
	Gain	**1.2**	**3.5**	**1.4**	**0.95**



III: There are numerous ways to improve `i-flow` in the near future.

- adjust hyperparameters
- use a CNN in the CL
- introduce Conditional Normalizing Flows or Discrete Flows to improve the multichannel or color sampling

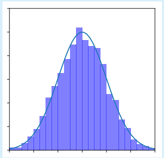
  Winkler et al. [1912.00042]; Tran et al. [1905.10347]
- “learn” the permutations: using 1×1 convolutions

  Kingma/Dhariwal [1807.03039]
- improve memory consumption with checkpointing

  Chen et al. [1604.06174]
- . . .

Event Generation with Normalizing Flows: `i-flow`

- I introduced the concepts of numerical integration and importance sampling.



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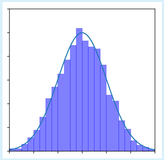
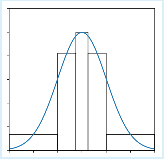


- I discussed "traditional" algorithms like, `VEGAS` or `Foam`.
- I compared ML approaches: learning $q(x)$ vs. GANs



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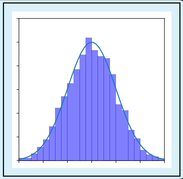

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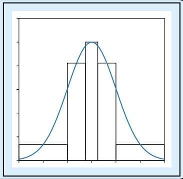
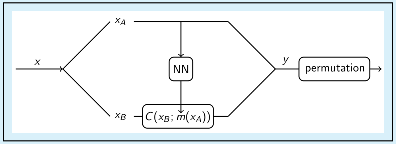


- I presented the idea of Normalizing Flows and showed their performance in test functions.
- I showed preliminary results for $pp \to W + nj$ with Sherpa.


x_A
x
NN
y
permutation
x_B
$C(x_B; m(x_A))$