Higgs Effective Field Theories and their Renormalization

— Theory Palaver, JGU Mainz —

Claudius Krause

Fermi National Accelerator Laboratory

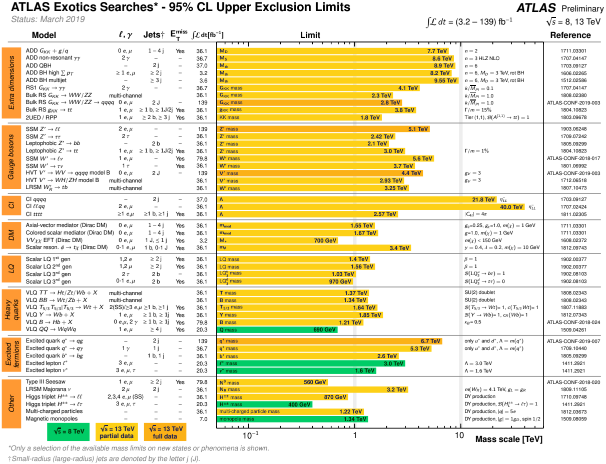
May 7, 2019

Unterstützt von / Supported by

Alexander von Humboldt Stiftung/Foundation

In collaboration with:
G. Buchalla, O. Catà, A. Celis, M. Knecht,
J. de Blas, O. Eberhardt, J. Toelstede

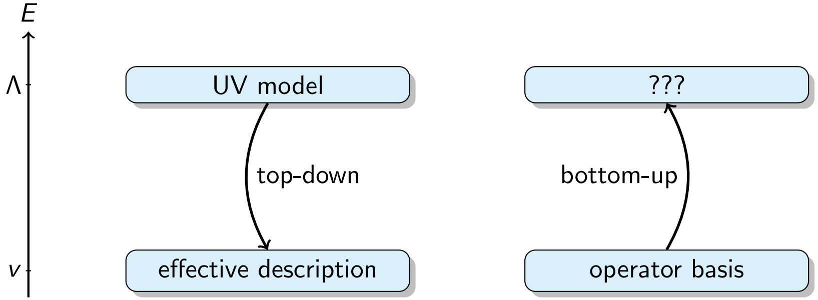
Where is the New Physics?



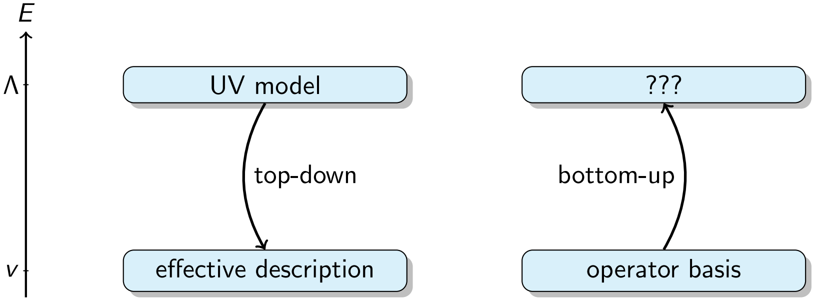
https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CombinedSummaryPlots/EXOTICS/

- The LHC confirmed the Standard Model.
- ATLAS [1207.7214] and CMS [1207.7235] found a Higgs-like scalar.
- All other searches for New Physics have been negative so far.

We can use EFTs because we have a mass gap.



We can use EFTs because we have a mass gap.



For a model-independent analysis we use the bottom-up approach.

However, we can learn something in both approaches:

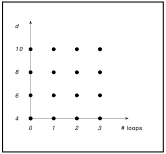
- top-down: Gain intuition with UV-models
- bottom-up: Fitting Wilson-coefficients

$\Rightarrow$ For a consistent analysis, we need the RGEs!

$\Rightarrow$ EFTs have become a popular field of research. $\Leftarrow$

Higgs Effective Field Theories and their Renormalization

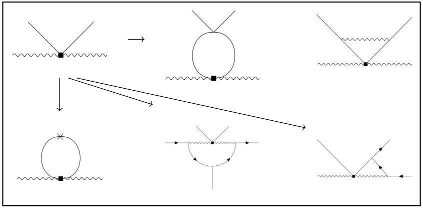
Part I: The two Higgs Effective Field Theories
[1307.5017,1412.6356,1803.00939]

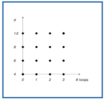


$$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$$

Part II: The Master Formula for 1-loop divergences
[1710.06412,1904.07840]

Part III: The Application to Higgs EFTs
[1710.06412,1904.07840]

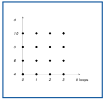




I: We distinguish 2 types of EFTs.

Ingredients:

- Particles: all SM particles (incl. 3 GBs for the $W^{\pm}/Z$ masses)
- Symmetries: $SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$, B, L
  at LO: flavor and custodial symmetry
- Power counting: depends on type of the EFT:



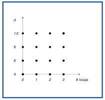
I: We distinguish 2 types of EFTs.

Ingredients:

- Particles: all SM particles (incl. 3 GBs for the $W^{\pm}/Z$ masses)
- Symmetries: $SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$, B, L
  at LO: flavor and custodial symmetry
- Power counting: depends on type of the EFT:

decoupling (linear) EFT:
– SMEFT –

- LO: SM
- Higgs is written as doublet ϕ
- expansion in canonical dimensions



I: We distinguish 2 types of EFTs.

Ingredients:

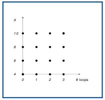
- Particles: all SM particles (incl. 3 GBs for the $W^{\pm}/Z$ masses)
- Symmetries: $SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$, B, L
  at LO: flavor and custodial symmetry
- Power counting: depends on type of the EFT:

decoupling (linear) EFT:
– SMEFT –

- LO: SM
- Higgs is written as doublet ϕ
- expansion in canonical dimensions

non-decoupling (nonlinear) EFT:
– $EWCh\mathcal{L}$ –

- LO: Higgs-less chiral Lagrangian + generic scalar h
- written in terms of U and h
- expansion in loops or chiral dimensions.

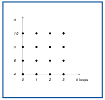


I: In the SMEFT, the expansion parameter is $\left(\frac{v}{\Lambda}\right)$.

Assumptions:

- There is a gap to the scale of new physics: $\Lambda \gg v$
- The low-energy field content contains the Higgs doublet.

$\rightarrow$ LO is renormalizable, new physics decouples. Appelquist/Carazzone ['75 PRD]



I: In the SMEFT, the expansion parameter is $\left(\frac{v}{\Lambda}\right)$.

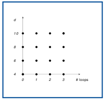
Assumptions:

- There is a gap to the scale of new physics: $\Lambda \gg v$
- The low-energy field content contains the Higgs doublet.

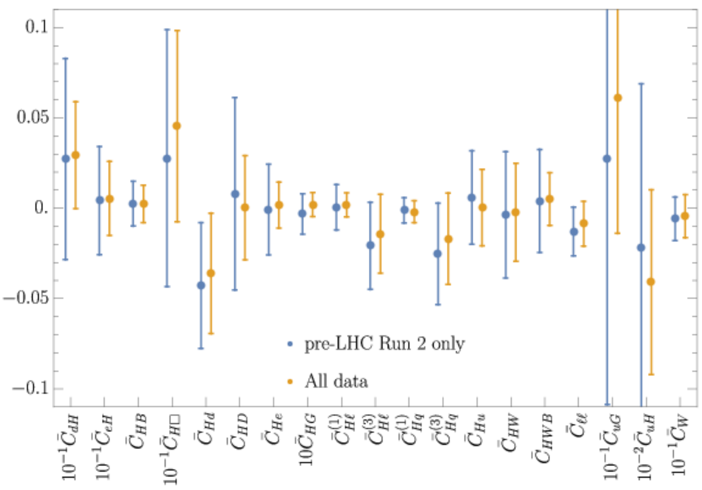
$\rightarrow$ LO is renormalizable, new physics decouples. Appelquist/Carazzone ['75 PRD]

$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{d=4} + \frac{v}{\Lambda}\mathcal{L}^{d=5} + \frac{v^2}{\Lambda^2}\mathcal{L}^{d=6} + \frac{v^3}{\Lambda^3}\mathcal{L}^{d=7} + \frac{v^4}{\Lambda^4}\mathcal{L}^{d=8} + \ldots$$

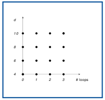
- dim 5: 2 operators (violating L) Weinberg ['79 PRL]
- dim 6: 76 operators (conserving B), 8 operators (violating B)
  Buchmüller, Wyler ['86 NPB]; Grzadkowski et al. [1008.4884, JHEP]
- dim 7: 30 operators (all violating L, some B)
  Lehman [1410.4193, PRD]; Liao/Ma [1607.07309, JHEP]
- dim 8: 895 operators (conserving B), 98 operators (violating B)
  Lehman/Martin [1510.00372, JHEP]; Henning et al. [1512.03433, JHEP]
  (For 1 generation and including hermitean conjugates.)



I: Current Data is consistent with the SM.



Ellis/Murphy/Sanz/You [1803.03252,JHEP]



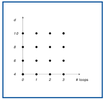
I: The construction of the chiral Lagrangian

Assumptions: Feruglio [hep-ph/9301281], Bagger *et al.* [hep-ph/9306256], Chivukula *et al.* [hep-ph/9312317], Wang/Wang [hep-ph/0605104], Grinstein/Trott[0704.1505], Contino[1005.4269], Alonso *et al.* [1212.3305], ...

- The pattern of symmetry breaking is $SU(2)_L \times SU(2)_R \to SU(2)_{V=L+R}$
- The transverse gauge bosons and the fermions of the SM are weakly coupled.

possibly:

- A new strong sector generates the 3 GBs of EWSB and the h at the scale f.



I: The construction of the chiral Lagrangian

Assumptions: Feruglio [hep-ph/9301281], Bagger *et al.* [hep-ph/9306256], Chivukula *et al.* [hep-ph/9312317], Wang/Wang [hep-ph/0605104], Grinstein/Trott[0704.1505], Contino[1005.4269], Alonso *et al.* [1212.3305], ...

- The pattern of symmetry breaking is $SU(2)_L \times SU(2)_R \to SU(2)_{V=L+R}$
- The transverse gauge bosons and the fermions of the SM are weakly coupled.

possibly:

- A new strong sector generates the 3 GBs of EWSB and the h at the scale f.

The Goldstone bosons φ are described by:

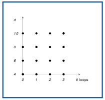
$$\mathcal{L} = \frac{v^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U \rangle, \qquad \text{where} \qquad U = \exp\left\{2i\frac{T_a\varphi_a}{v}\right\}.$$

Callan/Coleman/Wess/Zumino ['69 Phys. Rev.], Feruglio [hep-ph/9301281]

This was used in Chiral Perturbation Theory (χPT)

$$U \;\to\; lUr^\dagger, \qquad \text{where } l, r \in SU(3)_{L,R}$$

Gasser/Leutwyler ['84 Annals Phys., '85 Nucl. Phys. B]



I: The construction of the chiral Lagrangian

Assumptions: Feruglio [hep-ph/9301281], Bagger *et al.* [hep-ph/9306256], Chivukula *et al.* [hep-ph/9312317], Wang/Wang [hep-ph/0605104], Grinstein/Trott[0704.1505], Contino[1005.4269], Alonso *et al.* [1212.3305], ...

- The pattern of symmetry breaking is $SU(2)_L \times SU(2)_R \to SU(2)_{V=L+R}$
- The transverse gauge bosons and the fermions of the SM are weakly coupled.

possibly:

- A new strong ... ale f.

In unitary gauge:

$$\frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle = \frac{g^2 v^2}{4} W^+_\mu W^{\mu -} + \frac{(g^2+g'^2)v^2}{8} Z_\mu Z^\mu$$

The Goldstone bosons φ are de... by:

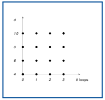
$$\mathcal{L} = \frac{v^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U\rangle, \qquad \text{where} \qquad U = \exp\left\{2i\frac{T_a\varphi_a}{v}\right\}.$$

Callan/Coleman/Wess/Zumino ['69 Phys. Rev.], Feruglio [hep-ph/9301281]

This was used in Chiral Perturbation Theory (χPT)

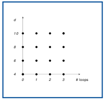
$$U \;\to\; lUr^\dagger, \qquad \text{where } l, r \in SU(3)_{L,R}$$

Gasser/Leutwyler ['84 Annals Phys., '85 Nucl. Phys. B]



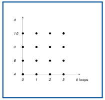
I: The construction of the chiral Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm LO} =& \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \ (1+F_U(h)) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}(h)\\
&+ i\bar{\Psi}_f \not{D}\Psi_f - (v\,\bar{\Psi}_f U\, Y_f(h)\Psi_f + \text{h.c.})\\
&- \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}
\end{aligned}
$$



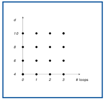
I: The construction of the chiral Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{LO}} =& \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \ (1+F_U(h)) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}(h)\\ &+ i\bar{\Psi}_f \not{D}\Psi_f - (v\,\bar{\Psi}_f U\, Y_f(h)\Psi_f + \mathrm{h.c.})\\ &- \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\end{aligned}$$



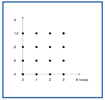
I: The construction of the chiral Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{LO}} = & \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \ (1 + F_U(h)) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}(h) \\
& + i\bar{\Psi}_f \not{D} \Psi_f - (v\,\bar{\Psi}_f U\, Y_f(h)\Psi_f + \text{h.c.}) \\
& - \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}
\end{aligned}
$$



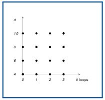
I: The construction of the chiral Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{LO}} = & \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \ (1 + F_U(h)) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}(h) \\
& + i\bar{\Psi}_f \not{D} \Psi_f - (v\,\bar{\Psi}_f U\, Y_f(h)\Psi_f + \text{h.c.}) \\
& - \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}
\end{aligned}
$$



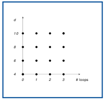
I: The construction of the chiral Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{LO}} = & \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \ (1+F_U(h)) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}(h) \\ & + i\bar{\Psi}_f \not{D}\Psi_f - (v\,\bar{\Psi}_f U\, Y_f(h)\Psi_f + \text{h.c.}) \\ & - \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\end{aligned}$$



I: The construction of the chiral Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{LO}} =& \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle\ (1+F_U(h)) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}(h)\\ &+ i\bar{\Psi}_f \not{D}\Psi_f - (v\,\bar{\Psi}_f U\, Y_f(h)\Psi_f + \mathrm{h.c.})\\ &- \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\end{aligned}$$

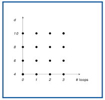


I: The construction of the chiral Lagrangian

$$\mathcal{L}_{\mathrm{LO}} = \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \; (1 + F_U(h)) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \mathcal{V}(h)$$
$$+ i\bar{\Psi}_f \not{D} \Psi_f - (v\,\bar{\Psi}_f U\, Y_f(h) \Psi_f + \mathrm{h.c.})$$
$$- \frac{1}{2}\langle G_{\mu\nu} G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu} W^{\mu\nu}\rangle - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Properties:

- It has generalized Higgs-couplings compared to the SM.
  $\Rightarrow$ related to the κ-formalism at LO.
- There is a hierarchy to the operators that modify the EWPD.
- It captures the low-energy effects of strongly-coupled new physics.
- It is non-renormalizable at LO.

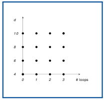


I: The Power counting is based on a loop expansion.

- $\mathcal{L}_{\text{LO}}$ is not renormalizable in the traditional sense, but it is renormalizable in the modern sense — order by order in an EFT:
- The LO counterterms are included at NLO.

$\Rightarrow$ The basis of NLO-operators is at least given by the counterterms of the one loop divergences.

- We identify $\frac{f^2}{\Lambda^2} \simeq \frac{1}{16\pi^2}$.
- The scale of new physics $f \approx v$, $\quad \xi = \frac{v^2}{f^2} \approx 1 \quad$ (to be relaxed later)



I: The Power counting is based on a loop expansion.

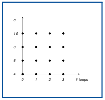
- $\mathcal{L}_{\text{LO}}$ is not renormalizable in the traditional sense, but it is renormalizable in the modern sense — order by order in an EFT:
- The LO counterterms are included at NLO.

$\Rightarrow$ The basis of NLO-operators is at least given by the counterterms of the one loop divergences.

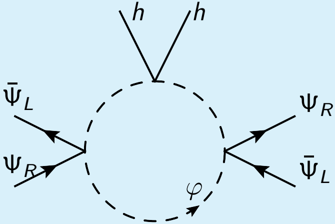
- We identify $\frac{f^2}{\Lambda^2} \simeq \frac{1}{16\pi^2}$.
- The scale of new physics $f \approx v$, $\quad \xi = \frac{v^2}{f^2} \approx 1 \quad$ (to be relaxed later)

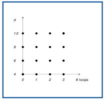
How can we identify the necessary counterterms?

- Using the superficial degree of divergence. $\Rightarrow$ next slide
- Computing all divergent one-loop terms. $\Rightarrow$ last part of this talk

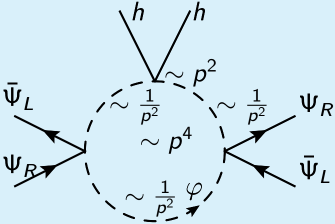


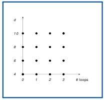
I: The Power counting is based on a loop expansion.



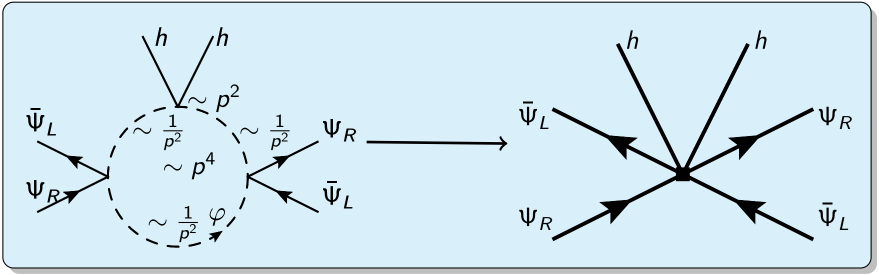


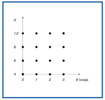
I: The Power counting is based on a loop expansion.



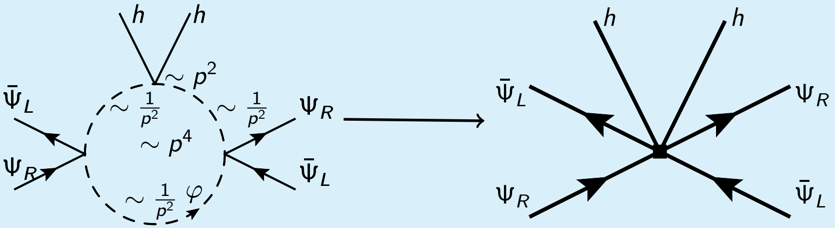


I: The Power counting is based on a loop expansion.

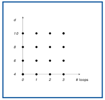




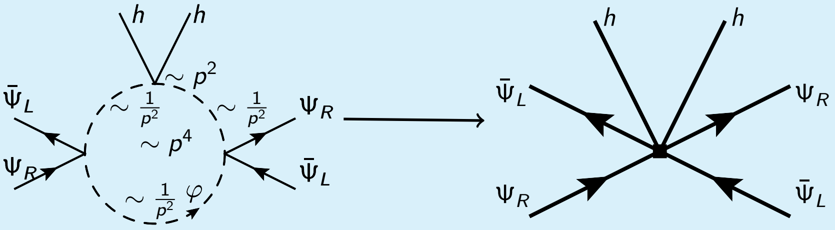
I: The Power counting is based on a loop expansion.



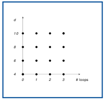
$$\mathcal{D} \sim p^{2L+2-X-\frac{1}{2}(F_L+F_R)-N_w} \left(\frac{\varphi}{v}\right)^B \left(\frac{h}{v}\right)^H \bar{\Psi}_L^{F_L^1} \Psi_L^{F_L^2} \bar{\Psi}_R^{F_R^1} \Psi_R^{F_R^2} \left(\frac{\mathcal{X}_{\mu\nu}}{v}\right)^X$$



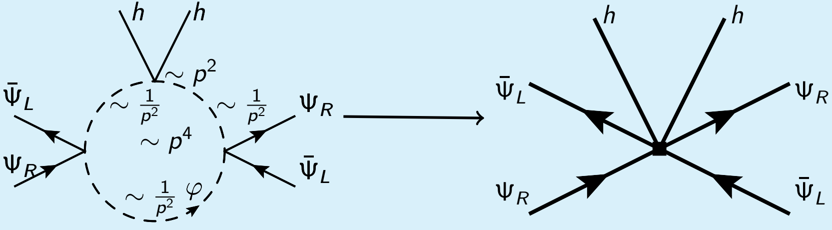
I: The Power counting is based on a loop expansion.



$$\mathcal{D} \sim p^{2L+2-X-\frac{1}{2}(F_L+F_R)-N_w} \left(\frac{\varphi}{v}\right)^B \left(\frac{h}{v}\right)^H \bar\Psi_L^{F_L^1}\, \Psi_L^{F_L^2}\, \bar\Psi_R^{F_R^1}\, \Psi_R^{F_R^2} \left(\frac{\mathcal{X}_{\mu\nu}}{v}\right)^X$$



I: The Power counting is based on a loop expansion.



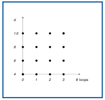
$$\mathcal{D} \sim p^{2L+2-X-\frac{1}{2}(F_L+F_R)-N_w} \left(\frac{\varphi}{v}\right)^B \left(\frac{h}{v}\right)^H \bar{\Psi}_L^{F_L^1}\, \Psi_L^{F_L^2}\, \bar{\Psi}_R^{F_R^1}\, \Psi_R^{F_R^2} \left(\frac{\mathcal{X}_{\mu\nu}}{v}\right)^X$$

This is equivalent to a counting of chiral dimensions:

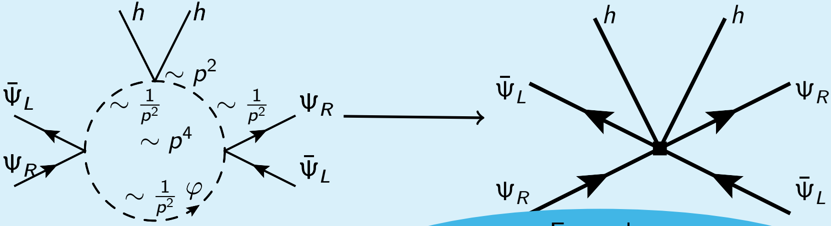
$$2L + 2 \;=\; [\text{couplings}]_\chi + [\text{derivatives}]_\chi + [\text{fields}]_\chi$$

$$[\text{bosons}]_\chi \;=\; 0,$$

$$[\text{fermion bilinears}]_\chi = [\text{derivatives}]_\chi = [\text{weak couplings}]_\chi = 1$$



I: The Power counting is based on a loop expansion.



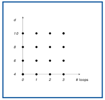
Example:
$[gg' B_{\mu\nu} \langle U T_3 U^\dagger W^{\mu\nu} \rangle \mathcal{F}(h)]_\chi = 4$
$\rightarrow L = 1$

$\mathcal{D} \sim p^{2L+2-X-\frac{1}{2}(F_L+F_R)-N_w} \left(\frac{\varphi}{v}\right)$

This is equivalent to a counting of chiral dimensions:

$$2L + 2 = [\text{couplings}]_\chi + [\text{derivatives}]_\chi + [\text{fields}]_\chi$$

$$[\text{bosons}]_\chi = 0,$$
$$[\text{fermion bilinears}]_\chi = [\text{derivatives}]_\chi = [\text{weak couplings}]_\chi = 1$$



I: Chiral dimensions have several applications.

- They are a generalization of the $\mathcal{O}(p)$-expansion of χPT:

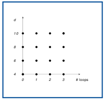
$\rightarrow$ Classify the NLO ($\chi = 4$) operators.

- Control the explicit breaking of symmetries (*e.g.* custodial or CP):
If they are broken by weak perturbations (like gauge or Yukawa), their spurions come with chiral dimensions as well.

- Gain additional informations about dimension 6 operators:
$[g^3 \langle W_\mu^\nu W_\nu^\rho W_\rho^\mu \rangle]_\chi = 6 \rightarrow$ arises at 2 loops
(given no states at f)

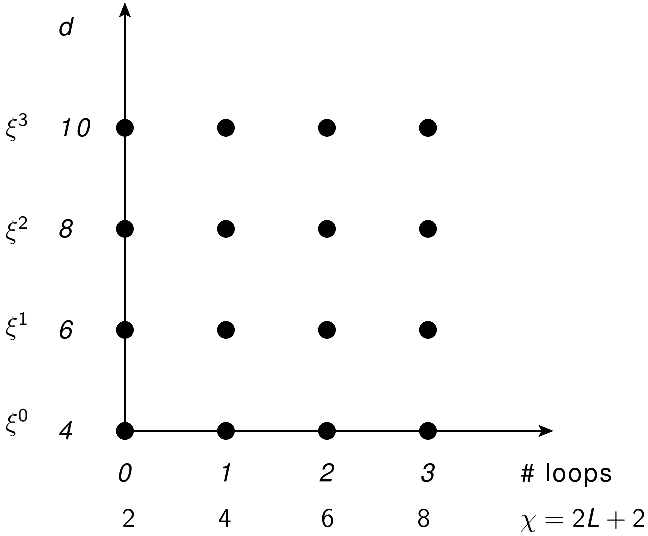
- They give the correct generalization of NDA, *i.e.* they also describe internal gauge and Yukawa lines.

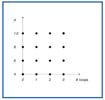
Naive Dimensional Analysis: Georgi, Manohar ['84 NPB]; Georgi [hep-ph/9207278]



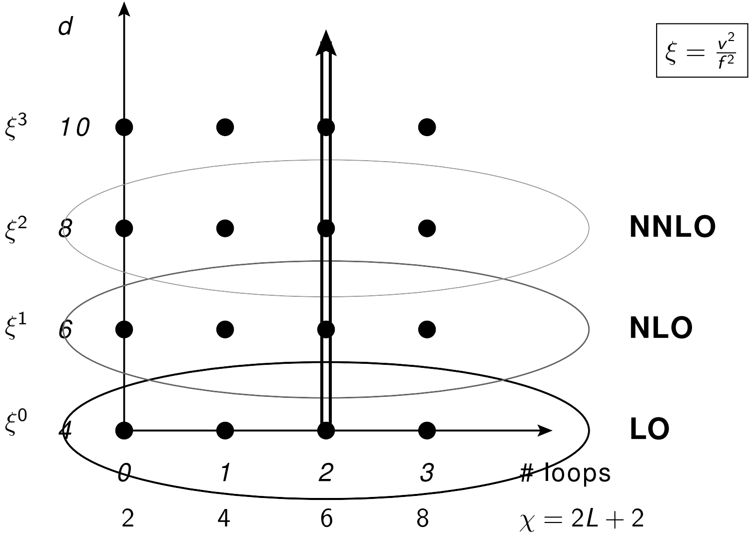
I: A graphical way to see the relation of SMEFT *vs.* *EWChℒ*

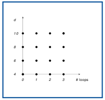
$$\boxed{\xi = \frac{v^2}{f^2}}$$



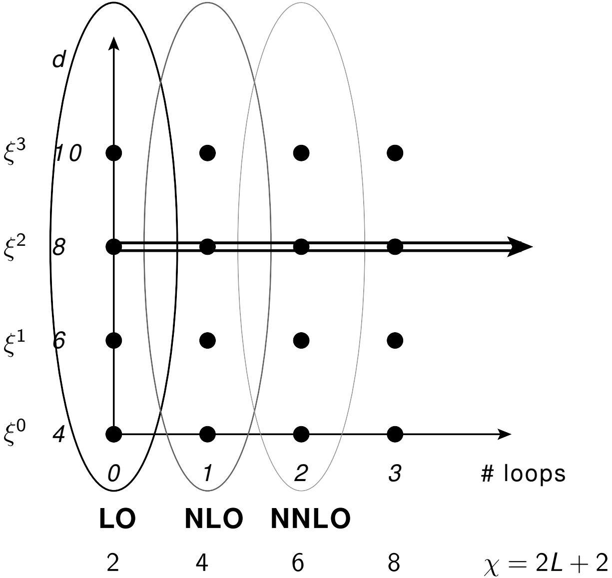


I: A graphical way to see the relation of SMEFT *vs. EWCh$\mathcal{L}$*

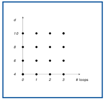




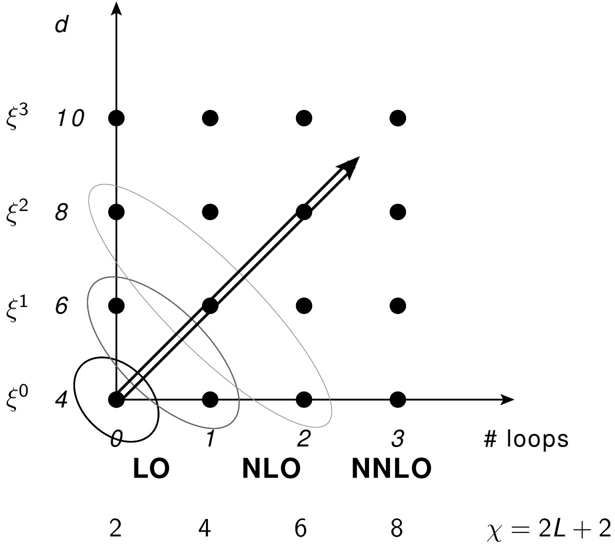
I: A graphical way to see the relation of SMEFT *vs.* *EWChℒ*



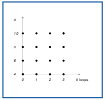
$\boxed{\xi = \frac{v^2}{f^2}}$



I: A graphical way to see the relation of SMEFT *vs.* *EWChℒ*



$$\boxed{\xi = \frac{v^2}{f^2}}$$



I: Current observables select $\mathcal{L}_{\text{fit}}$ from the *EWChL*.

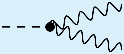
$$\mathcal{L}_{EWCh} = \mathcal{L}_{\text{kin}}^{h,\Psi,\text{gauge}} + \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle \ (1 + F_U(h)) - \mathcal{V}(h)$$
$$- (v\,\bar{\Psi}_f U\, Y_f(h)\Psi_f + \text{h.c.}) + \mathcal{L}_{\text{NLO}}$$

Buchalla/Catà/Celis/CK [1504.01707,NPB]

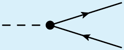
We focus on current observables and require $f > v$, *i.e.* $\xi = v^2/f^2 < 1$.

Single h processes:

tree:

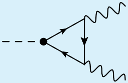


c_V



$c_{t,b,\tau,\mu,(c)}$

1-loop:

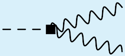


$c_{t,b,\tau,\mu,(c)}$

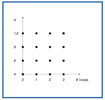
+

c_V

+



$c_{\gamma\gamma,gg,Z\gamma}$



I: Current observables select $\mathcal{L}_{\text{fit}}$ from the *EWChL*.

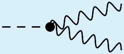
$$\begin{aligned}\mathcal{L}_{\text{fit}} =& 2c_V \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right)\left(\frac{h}{v}\right) \\ & - c_t y_t \bar{t} t h - c_b y_b \bar{b} b h - c_c y_c \bar{c} c h - c_\tau y_\tau \bar{\tau} \tau h - c_\mu y_\mu \bar{\mu} \mu h \\ & + \frac{e^2}{16\pi^2} c_{\gamma\gamma} F_{\mu\nu} F^{\mu\nu} \frac{h}{v} + \frac{e^2}{16\pi^2} c_{Z\gamma} Z_{\mu\nu} F^{\mu\nu} \frac{h}{v} + \frac{g_s^2}{16\pi^2} c_{gg} \langle G_{\mu\nu} G^{\mu\nu} \rangle \frac{h}{v}\end{aligned}$$

Buchalla/Catà/Celis/CK [1504.01707,NPB]

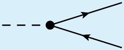
We focus on current observables and require $f > v$, *i.e.* $\xi = v^2/f^2 < 1$.

Single h processes:

tree:

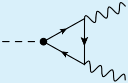


c_V



$c_{t,b,\tau,\mu,(c)}$

1-loop:

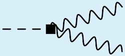


$c_{t,b,\tau,\mu,(c)}$

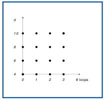
$+$

c_V

$+$



$c_{\gamma\gamma,gg,Z\gamma}$



I: Current observables select $\mathcal{L}_{\text{fit}}$ from the *EWCh$\mathcal{L}$*.

$$\begin{aligned}\mathcal{L}_{\text{fit}} = & 2c_V \left(m_W^2 W_\mu^+ W^{-\mu} + \tfrac{1}{2} m_Z^2 Z_\mu Z^\mu\right)\left(\tfrac{h}{v}\right)\\ & - c_t y_t \bar{t} t h - c_b y_b \bar{b} b h - c_c y_c \bar{c} c h - c_\tau y_\tau \bar{\tau}\tau h - c_\mu y_\mu \bar{\mu}\mu h\\ & + \frac{e^2}{16\pi^2} c_{\gamma\gamma} F_{\mu\nu} F^{\mu\nu} \frac{h}{v} + \frac{e^2}{16\pi^2} c_{Z\gamma} Z_{\mu\nu} F^{\mu\nu} \frac{h}{v} + \frac{g_s^2}{16\pi^2} c_{gg} \langle G_{\mu\nu} G^{\mu\nu}\rangle \frac{h}{v}\end{aligned}$$

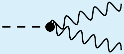
Buchalla/Catà/Celis/CK [1504.01707,NPB]

We focus on current observables and require $f > v$, *i.e.* $\xi = v^2/f^2 < 1$.

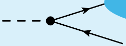
$c_i = \text{SM} + \mathcal{O}(\xi)$

Single h processes:

tree:

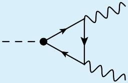


c_V



$c_{t,b,\tau,\mu,(c)}$

1-loop:

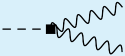


$c_{t,b,\tau,\mu,(c)}$

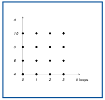
$+$

c_V

$+$

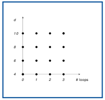


$c_{\gamma\gamma,gg,Z\gamma}$



I: We use HEPfit to find the current constraints.

HEPfit: ⇒ http://hepfit.roma1.infn.it/
A Code for the Combination of Indirect and Direct Constraints
on High Energy Physics Models. The HEPfit Collaboration [in preparation]



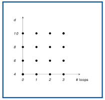
I: We use HEPfit to find the current constraints.

HEPfit: ⇒ http://hepfit.roma1.infn.it/
A Code for the Combination of Indirect and Direct Constraints on High Energy Physics Models.
The HEPfit Collaboration [in preparation]

It is:

- an open source fitter:
  available at https://github.com/silvest/HEPfit
- flexible:
  add your favorite model or observable
- a stand-alone code with few dependencies:
  ROOT, GSL, BOOST, (BAT)
- fast (& optional):
  using the MCMC implementation of the Bayesian Analysis Toolkit (BAT)

Caldwell/Kollar/Kroninger [0808.2552,Comput.Phys.Commun.]



I: We use HEPfit to find the current constraints.

Experimental input: For each decay channel we use the signal strength

$$\mu(Y) = \sum_X \text{eff}(X,Y) \frac{\sigma(X)\cdot \text{Br}\ (h\to Y)}{(\sigma(X)\cdot \text{Br}\ (h\to Y))_{\text{SM}}}$$

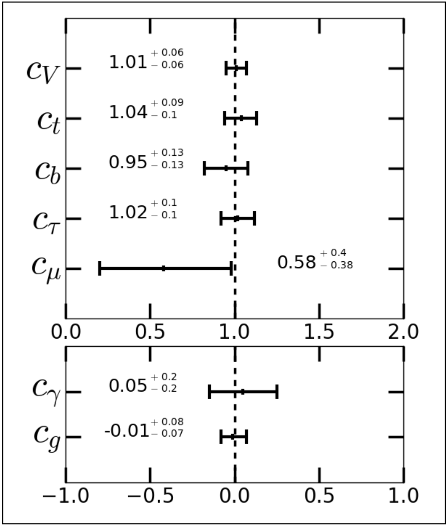
- If available, per experimental production category.
- Otherwise, per production mechanism.

		$b\bar{b}$	WW	$\tau\tau$	ZZ	$\gamma\gamma$	$Z\gamma$	$\mu\mu$
	SM Br	57.5%	21.6%	6.3%	2.7%	2.3‰	1.6‰	0.2‰
ggF8	87.2%	–	AC	AC	AC	AC	AC	AC
ggF13	87.1%	–	AC	C	AC	AC	AC	AC
VBF8	7.2%	–	AC	AC	AC	AC	AC	AC
VBF13	7.4%	C	AC	C	AC	AC	AC	AC
Vh8	5.1%	AC	AC	AC	AC	AC	AC	AC
Vh13	4.4%	AC	AC	C	AC	AC	AC	AC
tth8	0.6%	AC	–	–	AC	AC	AC	AC
tth13	1.0%	AC	AC	AC	AC	AC	AC	AC
Vh2		Tev	Uncertainty of the signal strengths $\mu \pm \Delta\mu$:					
tth2		Tev	$0 < \Delta\mu < 0.5$		$0.5 \leq \Delta\mu < 1.0$		$\Delta\mu > 1.0$	

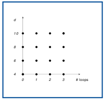
Table by Otto Eberhardt, HEFT '18, Mainz

I: The Posterior around the SM solution.

- The likelihood has multiple maxima ($c_i \to -c_i$ symmetries).
- We use a prior to select the SM-like solution.
- More details about the choice of priors are in [1803.00939,JHEP].
- Consistent with SM, but $\mathcal{O}(10\%)$ deviations still possible.
- $c_{Z\gamma}$ and c_c are not constrained beyond prior.



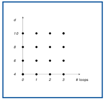
de Blas/Eberhardt/CK [1803.00939,JHEP]



I: Why should we go beyond tree level?

For consistent data analysis:

- Once we enter the precision phase at the LHC, loop effects cannot be neglected.
- LHC probes $E \sim v$, but new physics is matched to the EFT at $E \sim \Lambda \gg v$



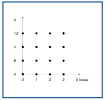
I: Why should we go beyond tree level?

For consistent data analysis:

- Once we enter the precision phase at the LHC, loop effects cannot be neglected.
- LHC probes $E \sim v$, but new physics is matched to the EFT at $E \sim \Lambda \gg v$

To learn more about the EFT itself:

- Are the bases of NLO operators complete?
- What subset of the chiral dimension 4 operators of the $EWCh\mathcal{L}$ are counterterms?



I: Why should we go beyond tree level?

For consistent data analysis:

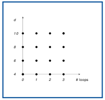
- Once we enter the precision phase at the LHC, loop effects cannot be neglected.
- LHC probes $E \sim v$, but new physics is matched to the EFT at $E \sim \Lambda \gg v$

To learn more about the EFT itself:

- Are the bases of NLO operators complete?
- What subset of the chiral dimension 4 operators of the $EWCh\mathcal{L}$ are counterterms?

To learn more about the field theory in general:

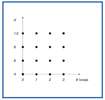
- How can we tackle an ∞ number of Feynman diagrams?



I: The two EFTs behave differently under renormalization.

SMEFT at 1 loop

- n LO vertices $\rightarrow$ LO operators
- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO ops.

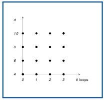


I: The two EFTs behave differently under renormalization.

SMEFT at 1 loop

- n LO vertices $\rightarrow$ LO operators
- $\left.\begin{matrix} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{matrix}\right\}$ LO + NLO ops.
- running of dim-4 altered by $\beta(c_{\text{SM}}) \sim \frac{\mu_\phi^2}{\Lambda^2}\, c_{\text{dim-6}}\, c_{\text{SM}}^m$
- running of dim-6 given by $\beta(c_{\text{dim-6}}) \sim c_{\text{dim-6}}\, c_{\text{SM}}^m$

$\Rightarrow$ NLO operators mix



I: The two EFTs behave differently under renormalization.

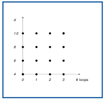
SMEFT at 1 loop

- n LO vertices $\to$ LO operators
- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO ops.
- running of dim-4 altered by $\beta(c_{\text{SM}}) \sim \frac{\mu_\phi^2}{\Lambda^2}\, c_{\text{dim-6}}\, c_{\text{SM}}^m$
- running of dim-6 given by $\beta(c_{\text{dim-6}}) \sim c_{\text{dim-6}}\, c_{\text{SM}}^m$

$\Rightarrow$ NLO operators mix

$EWCh\mathcal{L}$ at 1 loop

- n LO vertices $\to$ LO+NLO ops.
- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO + NNLO ops.



I: The two EFTs behave differently under renormalization.

SMEFT at 1 loop

- n LO vertices $\to$ LO operators
- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO ops.
- running of dim-4 altered by $\beta(c_{\mathrm{SM}}) \sim \frac{\mu_\phi^2}{\Lambda^2}\, c_{\text{dim-6}}\, c_{\mathrm{SM}}^m$
- running of dim-6 given by $\beta(c_{\text{dim-6}}) \sim c_{\text{dim-6}}\, c_{\mathrm{SM}}^m$

$\Rightarrow$ NLO operators mix

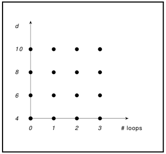
***EWChL* at 1 loop** ($EWCh\mathcal{L}$)

- n LO vertices $\to$ LO+NLO ops.
- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO + NNLO ops.
- running of LO given by $\beta(c_{\mathrm{LO}}) \sim c_{\mathrm{LO}}^m$
- running of NLO given by $\beta(c_{\mathrm{NLO}}) \sim c_{\mathrm{LO}}^m$

$\Rightarrow$ NLO operators do not mix

Higgs Effective Field Theories and their Renormalization

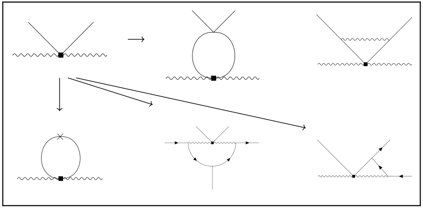
Part I: The two Higgs Effective Field Theories
[1307.5017,1412.6356,1803.00939]



$$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$$

Part II: The Master Formula
for 1-loop divergences
[1710.06412,1904.07840]

Part III: The Application
to Higgs EFTs
[1710.06412,1904.07840]



II: We use the Background-Field-Method...

starting from the generating functional:

$$Z[j,\rho,\bar{\rho}] = e^{iW[j,\rho,\bar{\rho}]} = \int [d\phi d\psi d\bar{\psi}] \quad e^{i(S[\phi,\psi,\bar{\psi}]+j\phi+\bar{\psi}\rho+\bar{\rho}\psi)},$$

$$\phi = \hat{\phi} + \phi_{qu}, \qquad \psi = \hat{\psi} + \psi_{qu},$$

$$\Rightarrow e^{iW_{L=1}} = \int [d\phi_{qu} d\psi_{qu} d\bar{\psi}_{qu}] \quad e^{iS^{(2)}[\hat{\phi},\hat{\psi},\hat{\bar{\psi}};\phi_{qu},\psi_{qu},\bar{\psi}_{qu}]}$$

Abbott '81

Quantum gauge fixing:

$$\mathcal{L}_{\text{gauge-fix}} = -\frac{1}{2\xi}\left(\partial_\mu B^\mu + \frac{\xi}{2} g' v \varphi_3\right)^2 - \frac{1}{\xi}\,\mathrm{Tr}\left\{\left(\hat{D}^\mu_W W_\mu - \frac{\xi}{2} g v \hat{U} \varphi \hat{U}^\dagger\right)^2\right\}$$

- The terms proportional to φ will make the next steps easier.
- Later, we will set $\xi = 1$.

Dittmaier/Grosse-Knetter [hep-ph/9505266,NPB]

$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$ II: ... and Super-Heat-Kernel Expansion...

evaluating the one-loop functional — Neufeld/Gasser/Ecker [hep-ph/9806436,PLB]

$$e^{iW_{L=1}} = \int [d\phi d\psi d\bar{\psi}] \quad e^{iS^{(2)}[\hat{\phi},\hat{\psi},\hat{\bar{\psi}};\phi,\psi,\bar{\psi}]}$$

$$S^{(2)} = \frac{1}{2}\phi A \phi + \bar{\psi} B \psi + \phi \bar{\Gamma} \psi + \bar{\psi} \Gamma \phi$$

$$W_{L=1} = \frac{i}{2} \operatorname{Tr} \ln A - i \operatorname{Tr} \ln B - \frac{i}{2} \sum_{n=0}^{\infty} \frac{1}{n} \operatorname{Tr} \left(A^{-1} \bar{\Gamma} B^{-1} \Gamma - A^{-1} \Gamma^T B^{-1,T} \bar{\Gamma}^T \right)^n$$

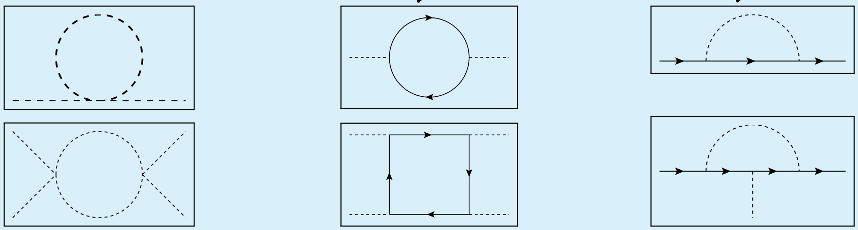
$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$ II: ... and Super-Heat-Kernel Expansion...

evaluating the one-loop functional — Neufeld/Gasser/Ecker [hep-ph/9806436,PLB]

$$e^{iW_{L=1}} = \int [d\phi d\psi d\bar{\psi}] \quad e^{iS^{(2)}[\hat{\phi},\hat{\psi},\hat{\bar{\psi}};\phi,\psi,\bar{\psi}]}$$

$$S^{(2)} = \frac{1}{2}\phi A \phi + \bar{\psi} B \psi + \phi \bar{\Gamma} \psi + \bar{\psi} \Gamma \phi$$

$$W_{L=1} = \frac{i}{2} \mathrm{Tr} \ln A - i \,\mathrm{Tr} \ln B - \frac{i}{2} \sum_{n=0}^{\infty} \frac{1}{n} \mathrm{Tr} \left(A^{-1} \bar{\Gamma} B^{-1} \Gamma - A^{-1} \Gamma^T B^{-1,T} \bar{\Gamma}^T \right)^n$$



Neufeld/Gasser/Ecker [hep-ph/9806436,PLB]

Introducing supermatrix algebra:

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{aligned} \operatorname{Sdet} M &= \det\left(a - bd^{-1}c\right)\det d^{-1} \\ \operatorname{Str} M &= \operatorname{Tr} a - \operatorname{Tr} d \\ \operatorname{Sdet} M &= e^{\operatorname{Str}\ln M} \end{aligned}$$

$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$ II: ... and Super-Heat-Kernel Expansion...

Neufeld/Gasser/Ecker [hep-ph/9806436,PLB]

Introducing supermatrix algebra:

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{aligned} \mathrm{Sdet}\, M &= \det\left(a - bd^{-1}c\right)\det d^{-1} \\ \mathrm{Str}\, M &= \mathrm{Tr}\, a - \mathrm{Tr}\, d \\ \mathrm{Sdet}\, M &= e^{\mathrm{Str}\ln M} \end{aligned}$$

The one-loop functional of $S^{(2)} = \frac{1}{2}\phi A\phi + \bar{\psi}B\psi + \phi\bar{\Gamma}\psi + \bar{\psi}\Gamma\phi$ becomes:

$$W_{L=1} = \frac{i}{2}\,\mathrm{Str}\ln\Delta, \qquad \Delta = \begin{pmatrix} A & \bar{\Gamma} & -\Gamma^T \\ -\bar{\Gamma}^T & 0 & -B^T \\ \Gamma & B & 0 \end{pmatrix}$$

$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$ II: ... and Super-Heat-Kernel Expansion...

Applying the Heat-Kernel Expansion:

Donoghue/Golowich/Holstein '92

Neufeld/Gasser/Ecker hep-ph/9806436

$$\begin{aligned} W_{L=1} &= \frac{i}{2}\,\mathrm{Str}\ln\Delta \\ &= -\frac{i}{2}\int_0^\infty \frac{d\tau}{\tau}\int d^dx\,\mathrm{str}\,\langle x|e^{-\tau\Delta}|x\rangle \end{aligned}$$

with the expansion in Seeley-DeWitt coefficients

$$\langle x|e^{-\tau\Delta}|x\rangle = \frac{i}{(4\pi)^{d/2}}\frac{e^{-\tau m^2}}{\tau^{d/2}}\sum_{n=0}^{\infty} a_n(x)\tau^n$$

Applying the Heat-Kernel Expansion:

Donoghue/Golowich/Holstein '92

Neufeld/Gasser/Ecker hep-ph/9806436

$$W_{L=1} = \frac{i}{2} \operatorname{Str} \ln \Delta$$
$$= -\frac{i}{2} \int_0^\infty \frac{d\tau}{\tau} \int d^d x \operatorname{str} \langle x | e^{-\tau\Delta} | x \rangle$$

with the expansion in Seeley-DeWitt coefficients

$$\langle x | e^{-\tau\Delta} | x \rangle = \frac{i}{(4\pi)^{d/2}} \frac{e^{-\tau m^2}}{\tau^{d/2}} \sum_{n=0}^{\infty} a_n(x) \tau^n$$

- The a_n can be computed, knowing the form of Δ.
- The UV-divergences of $W_{L=1}$ are the poles in $\frac{1}{\tau}$.
  $\Rightarrow$ only a_2 contributes!

$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$ II: ... and Super-Heat-Kernel Expansion...

The Heat-Kernel Expansion extracts the $\frac{1}{\epsilon}$-poles of $W_{L=1}$.

$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$ II: ... and Super-Heat-Kernel Expansion...

The Heat-Kernel Expansion extracts the $\frac{1}{\epsilon}$-poles of $W_{L=1}$.

With Donoghue/Golowich/Holstein '92; Neufeld/Gasser/Ecker [hep-ph/9806436,PLB]

$$\Delta = (\partial_\mu + \Lambda_\mu)(\partial^\mu + \Lambda^\mu) + \Sigma$$

we get

$$W_{L=1,div} = \frac{1}{32\pi^2\epsilon}\int d^4x \,\mathrm{str}\left[\frac{1}{12}\Lambda_{\mu\nu}\Lambda^{\mu\nu} + \frac{1}{2}\Sigma\Sigma\right].$$

$$\Lambda_{\mu\nu} = \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu + [\Lambda_\mu, \Lambda_\nu]$$

- Specifying the Dirac structure of $S^{(2)}$, we can further evaluate the Dirac-traces.
- The resulting Master-Formula is purely algebraic (Matrix multiplication and traces). 'tHooft '73,NPB

$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$ II: ... to find the Master Formula.

In the SM (and the *EWChL*), we have

$$\mathcal{L}_2^{\text{SM}} = -\frac{1}{2}\phi^i A_i^j \phi_j + \bar{\chi}\left(i\partial\!\!\!/ - G\right)\chi + \bar{\chi}\Gamma^i\phi_i + \phi^i\bar{\Gamma}_i\chi,$$

with $A = (\partial^\mu + N^\mu)(\partial_\mu + N_\mu) + Y$ and $G \equiv (r + \rho_\mu\gamma^\mu)P_R + (l + \lambda_\mu\gamma^\mu)P_L$.

This gives

$$\mathcal{L}_{\text{div}}^{\text{SM}} = \frac{1}{32\pi^2\varepsilon}\left(\text{tr}\left[\frac{1}{12}N^{\mu\nu}N_{\mu\nu} + \frac{1}{2}Y^2 - \frac{1}{3}\left(\lambda^{\mu\nu}\lambda_{\mu\nu} + \rho^{\mu\nu}\rho_{\mu\nu}\right)\right]\right.$$
$$\left. + \,\text{tr}\left[2D^\mu l D_\mu r - 2lrlr\right] + \bar{\Gamma}\left(i\partial\!\!\!/ + iN\!\!\!/ + \frac{1}{2}\gamma^\mu G\gamma_\mu\right)\Gamma\right)$$

with

$$N_{\mu\nu} \equiv \partial_\mu N_\nu - \partial_\nu N_\mu + [N_\mu, N_\nu]\,,$$
$$\lambda_{\mu\nu} \equiv \partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu + i[\lambda_\mu, \lambda_\nu]\,, \qquad \rho_{\mu\nu} \equiv \partial_\mu\rho_\nu - \partial_\nu\rho_\mu + i[\rho_\mu, \rho_\nu]\,,$$
$$D_\mu l \equiv \partial_\mu l + i\rho_\mu l - il\lambda_\mu\,, \qquad D_\mu r \equiv \partial_\mu r + i\lambda_\mu r - ir\rho_\mu\,.$$

'tHooft '73,NPB; Buchalla/Catà/Celis/Knecht/CK [1710.06412,NPB]

$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$ II: ... to find the Master Formula.

For the bosonic SMEFT operators in the Warsaw Basis, we have

$$\mathcal{L}_2^{\text{SMEFT}} = \mathcal{L}_2^{\text{SM}} + \phi^i \left(a^{ij}_{\mu\nu} D^\mu D^\nu + 2 b^{ij}_\mu D^\mu + c^{ij} \right) \phi_j,$$

with $a_{\mu\nu}, b_\mu, c \sim \frac{1}{\Lambda^2}$.

Warsaw Basis: Grzadkowski/Iskrzynski/Misiak/Rosiek [1008.4884,JHEP]

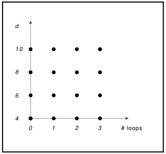
This gives

$$\begin{aligned}\mathcal{L}_{\text{div}}^{\text{SMEFT}} = \mathcal{L}_{\text{div}}^{\text{SM}} + \frac{1}{32\pi^2\varepsilon} \Bigg(& \text{tr}\left[cY + \frac{1}{3} N_{\mu\nu}[D^\mu, b^\nu] + i\bar{\Gamma} \not{b} \Gamma - \frac{1}{6} \bar{\Gamma} i \overleftrightarrow{\not{D}} \not{a} \Gamma \right] \\ & + \text{tr}\left[\frac{1}{6} a^{\mu\nu} N_{\mu\lambda} N_\nu{}^\lambda - \frac{1}{24} a^\lambda_\lambda N_{\mu\nu} N^{\mu\nu} + \frac{1}{6} N_{\mu\lambda} [D_\nu, [D^\lambda, a^{\mu\nu}]] \right] \\ & + \text{tr}\left[\frac{1}{3} Y[D_\mu, [D_\nu, a^{\mu\nu}]] - \frac{1}{4} a^\lambda_\lambda Y^2 - \frac{1}{12} Y[D_\mu, [D^\mu, a^\lambda_\lambda]] \right] \Bigg).\end{aligned}$$

Buchalla/Celis/CK/Toelstede [1904.07840]

Higgs Effective Field Theories and their Renormalization

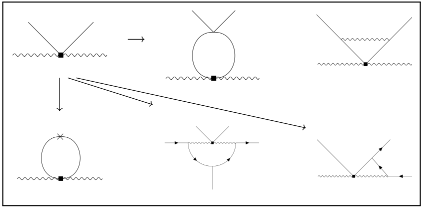
Part I: The two Higgs Effective Field Theories
[1307.5017,1412.6356,1803.00939]

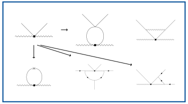


$$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$$

Part II: The Master Formula
for 1-loop divergences
[1710.06412,1904.07840]

Part III: The Application
to Higgs EFTs
[1710.06412,1904.07840]





III: The two EFTs behave differently under renormalization.

SMEFT at 1 loop

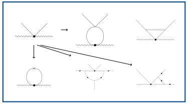
- n LO vertices $\rightarrow$ LO operators
- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO ops.
- running of dim-4 altered by $\beta(c_{\text{SM}}) \sim \frac{\mu_\phi^2}{\Lambda^2}\, c_{\text{dim-6}}\, c_{\text{SM}}^m$
- running of dim-6 given by $\beta(c_{\text{dim-6}}) \sim c_{\text{dim-6}}\, c_{\text{SM}}^m$

$\Rightarrow$ NLO operators mix

EWCh$\mathcal{L}$ at 1 loop

- n LO vertices $\rightarrow$ LO+NLO ops.
- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO + NNLO ops.
- running of LO given by $\beta(c_{\text{LO}}) \sim c_{\text{LO}}^m$
- running of NLO given by $\beta(c_{\text{NLO}}) \sim c_{\text{LO}}^m$

$\Rightarrow$ NLO operators do not mix

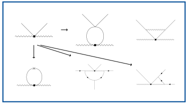


III: Renormalization of the SMEFT

Buchalla/Celis/CK/Toelstede [1904.07840]

Cross checks:

- We reproduce the results of the bosonic sector of Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014,JHEP]
- We performed 3.x independent computations.



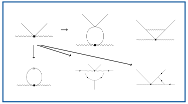
III: Renormalization of the SMEFT

Cross checks: Buchalla/Celis/CK/Toelstede [1904.07840]

- We reproduce the results of the bosonic sector of Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014,JHEP]
- We performed 3.x independent computations.

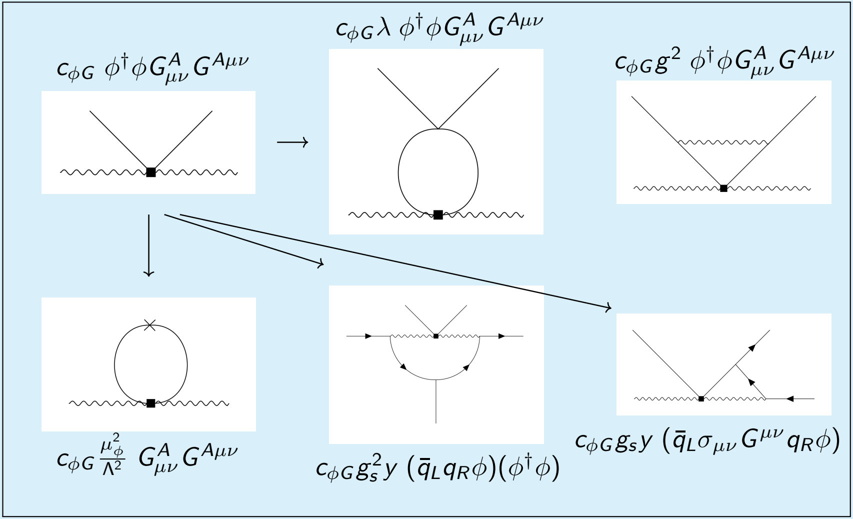
The result:

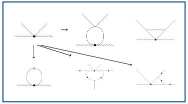
- suggests the completeness of the NLO basis. Grzadkowski/Iskrzynski/Misiak/Rosiek [1008.4884,JHEP]
- confirms the running and mixing of the NLO operators. Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014,JHEP]



III: Renormalization of the SMEFT

example of operator mixing: $\mathcal{O}_{\phi G} = \phi^\dagger \phi G^A_{\mu\nu} G^{A\mu\nu}$





III: The two EFTs behave differently under renormalization.

SMEFT at 1 loop

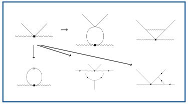
- n LO vertices $\rightarrow$ LO operators
- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO ops.
- running of dim-4 altered by $\beta(c_{\text{SM}}) \sim \frac{\mu_\phi^2}{\Lambda^2} c_{\text{dim-6}}\, c_{\text{SM}}^m$
- running of dim-6 given by $\beta(c_{\text{dim-6}}) \sim c_{\text{dim-6}}\, c_{\text{SM}}^m$

$\Rightarrow$ NLO operators mix

EWCh$\mathcal{L}$ at 1 loop

- n LO vertices $\rightarrow$ LO+NLO ops.
- $\left.\begin{array}{l} n \text{ LO vertices} \\ 1 \text{ NLO vertex} \end{array}\right\}$ LO + NLO + NNLO ops.
- running of LO given by $\beta(c_{\text{LO}}) \sim c_{\text{LO}}^m$
- running of NLO given by $\beta(c_{\text{NLO}}) \sim c_{\text{LO}}^m$

$\Rightarrow$ NLO operators do not mix



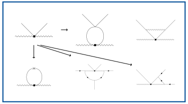
III: Renormalization of the *EWCh$\mathcal{L}$*

Cross checks: Buchalla/Catà/Celis/Knecht/CK [1710.06412,NPB]

- We reproduce previous results of the Scalar sector.

  Guo/Ruiz-Femenía/Sanz-Cillero, Phys. Rev. D **92** (2015) 074005, arXiv:1506.04204

- We reproduce the SM-β-functions in the SM-limit.
- We performed 4.x independent computations with 2 different choices of $\mathcal{L}_{\text{gauge-fix}}$.



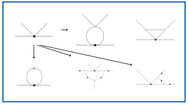
III: Renormalization of the *EWCh$\mathcal{L}$*

Cross checks: Buchalla/Catà/Celis/Knecht/CK [1710.06412,NPB]

- We reproduce previous results of the Scalar sector.
  Guo/Ruiz-Femenía/Sanz-Cillero, Phys. Rev. D **92** (2015) 074005, arXiv:1506.04204
- We reproduce the SM-β-functions in the SM-limit.
- We performed 4.x independent computations with 2 different choices of $\mathcal{L}_{\text{gauge-fix}}$.

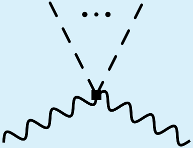
The result: see also Alonso *et al.*, arXiv:1710.06848, PRD

- confirms the predictions by power counting.
  Buchalla/Catà/CK, Phys. Lett. B **731** (2014) 80, arXiv:1312.5624
- is consistent with the operator basis.
  Buchalla/Catà/CK, Nucl. Phys. B **880** (2014) 552, arXiv:1307.5017



III: Renormalization of the *EWChℒ*

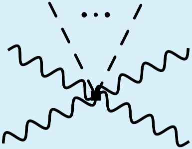
$g^2 U D^2 h$



$$\mathcal{O}_\beta = (g' v)^2 \langle U T_3 D_\mu U^\dagger \rangle^2 \mathcal{F},$$

1/1 operator, $\sim (F_U - F_U'^2/4)$

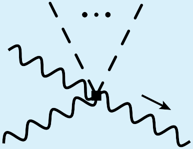
$U D^4 h$



$$\mathcal{O}_{D1} = \langle D_\mu U D^\mu U^\dagger \rangle^2 \mathcal{F},$$

5/15 operators generated

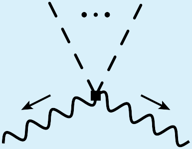
$g U X D^2 h$



$$\mathcal{O}_{XU7} = g' \langle T_3 D_\mu U^\dagger D_\nu U \rangle B^{\mu\nu} \bar{\mathcal{F}},$$

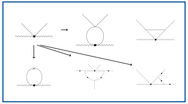
0/8 operators generated

$g^2 U X^2 h$



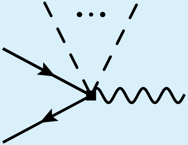
$$\mathcal{O}_{Xh1} = g'^2 B_{\mu\nu} B^{\mu\nu} \bar{\mathcal{F}},$$

0/10 operators, (3 op. $\mathcal{F}(h) = \text{const.} \Rightarrow \mathcal{L}_{LO}$)



III: Renormalization of the $EWCh\mathcal{L}$

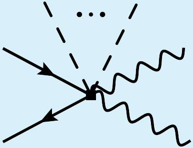
$y^2 U D \Psi^2 h$



$$\mathcal{O}_{\Psi V1} = i y^2 (\bar{q}_L \gamma^\mu q_L) \langle U T_3 D_\mu U^\dagger \rangle \, \mathcal{F},$$

13/13 operators generated

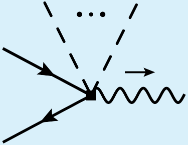
$y U D^2 \Psi^2 h$



$$\mathcal{O}_{\Psi S1/2} = y \bar{q}_L U P_\pm q_R \langle D_\mu U D^\mu U^\dagger \rangle \, \mathcal{F},$$

12/30 operators generated (+h.c.)

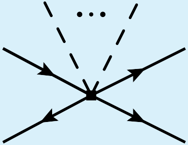
$y g U \Psi^2 X h$



$$\mathcal{O}_{\Psi X1/2} = y g' \bar{q}_L \sigma_{\mu\nu} U P_\pm q_R B^{\mu\nu} \, \mathcal{F},$$

0/11 operators generated (+h.c.)

$y^2 \Psi^4 U h$

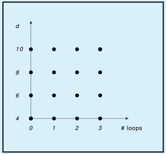


$$\mathcal{O}_{LL1} = y^2 (\bar{q}_L \gamma^\mu q_L)(\bar{q}_L \gamma_\mu q_L) \, \mathcal{F},$$

22/60 operators (+h.c.), from $Y \cdot Y$ or $Y^a \cdot Y^a$

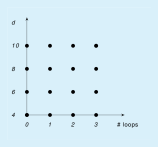
Higgs Effective Field Theories and their Renormalization — Summary —

- I introduced two EFTs for physics beyond the SM: the (decoupling) SMEFT and the (nondecoupling) $EWCh\mathcal{L}$
- I showed how the two EFTs are related to each other.



Higgs Effective Field Theories and their Renormalization — Summary —

- I introduced two EFTs for physics beyond the SM: the (decoupling) SMEFT and the (nondecoupling) *EWChℒ*
- I showed how the two EFTs are related to each other.



- I derived a master formula for the $1/\epsilon$-poles of a given Lagrangian, based on the super-heat-kernel.

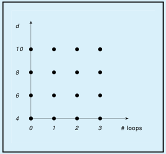
$$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$$

- The result is purely algebraic (matrix multiplication and -tracing).

Higgs Effective Field Theories and their Renormalization — Summary —

- I introduced two EFTs for physics beyond the SM: the (decoupling) SMEFT and the (nondecoupling) *EWChL*

- I showed how the two EFTs are related to each other.

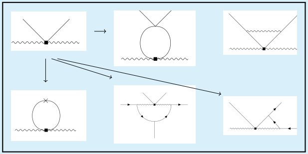


- I derived a master formula for the $1/\epsilon$-poles of a given Lagrangian, based on the super-heat-kernel.

$$\boxed{\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2}$$

- The result is purely algebraic (matrix multiplication and -tracing).

- For the SMEFT, we confirmed the RGE results of the bosonic sector.
- For the $EWCh\mathcal{L}$, we computed the 1-loop RGEs and confirmed the NLO basis.



Backup

Naive Dimensional Analysis (NDA)

Naive dimensional analysis - NDA:

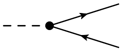
Georgi, Manohar ['84 NPB]; Georgi [hep-ph/9207278]

- Overall factor $f^2\Lambda^2$, f^{-1} for each strongly interacting field, Λ^{-1} to reach dimension 4.
- It is consistent with our counting only if internal gauge lines and Yukawa interactions are neglected.
- It gives a wrong scaling in some cases, *e.g.* $F_{\mu\nu}F^{\mu\nu}$.

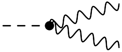
There is a relation between the electroweak chiral Lagrangian and the κ framework.

EWChL

tree:

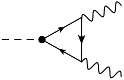


$c_{t,b,\tau}$

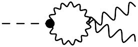


c_V

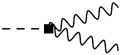
1-loop:



$c_{t,b,\tau}$ +



c_V +

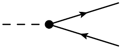


$c_{\gamma\gamma,gg}$

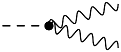
$\kappa_i^2 = \Gamma^i/\Gamma^i_{\mathrm{SM}}, \quad \kappa_i^2 = \sigma^i/\sigma^i_{\mathrm{SM}}$

LHCHXSWG [1209.0040,1307.1347]

tree:

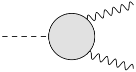


$\kappa_{t,b,\tau}$



κ_V

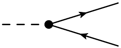
1-loop:

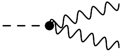


$\kappa_{\gamma,g}$

There is a relation between the electroweak chiral Lagrangian and the κ framework.

EWChL



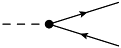


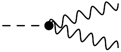
tree:

$c_{t,b,\tau}$ c_V

$$\kappa_i^2 = \Gamma^i/\Gamma^i_{\mathrm{SM}}, \quad \kappa_i^2 = \sigma^i/\sigma^i_{\mathrm{SM}}$$

LHCHXSWG [1209.0040,1307.1347]





tree:

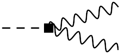
$\kappa_{t,b,\tau}$ κ_V

Both have the same number of free parameters:

$$\{c_V, c_{t,b,\tau}, c_\gamma, c_g\} \quad vs. \quad \{\kappa_V, \kappa_{t,b,\tau}, \kappa_\gamma, \kappa_g\}$$

$\Rightarrow$ κ's are QFT consistent and with small modifications directly interpretable within an EFT!

1-lo



$c_{\gamma\gamma,gg}$

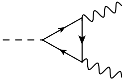
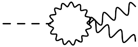
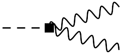
$\kappa_{\gamma,g}$

The κ framework cannot be recovered as a limit of the SMEFT (dim 6).

Full dimension 6 Grzadkowski *et al.* [1008.4884, JHEP]:

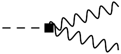
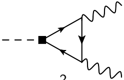
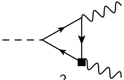
example: $h \to Z\gamma$

LO:

 $+$  $+$  $+ \quad \ldots$

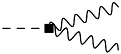
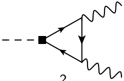
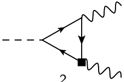
SM $+$ SM $+$ $\mathcal{O}(\frac{v^2}{\Lambda^2})$

LO + NLO:

 $+$  $+$  $+ \quad \ldots$

SM $+ \mathcal{O}(\frac{v^2}{\Lambda^2})$ $+$ $\mathcal{O}(\frac{v^2}{16\pi^2\Lambda^2})$ $+$ $\mathcal{O}(\frac{v^2}{16\pi^2\Lambda^2})$

Additional assumption of weakly coupled UV Einhorn/Wudka[1307.0478]:

 $+$  $+$  $+ \quad \ldots$

SM $+ \mathcal{O}(\frac{v^2}{16\pi^2\Lambda^2})$ $+$ $\mathcal{O}(\frac{v^2}{16\pi^2\Lambda^2})$ $+$ $\mathcal{O}(\frac{v^2}{16\pi^2\Lambda^2})$

The Minimal Composite Higgs Model

Agashe *et al.* [hep-ph/0412089,NPB], Contino *et al.* [hep-ph/0612048,PRD]

- global symmetry spontaneoulsy broken at scale f: $SO(5) \to SO(4)$
- $SU(2)_L \times U(1)_Y \subset SO(4)$ is gauged

$\to$ massive $W^\pm/Z$, light h

$$\mathcal{L}_{\text{kin}} = \frac{f^2}{2}(D_\mu\Sigma)^T(D^\mu\Sigma),$$

where

$$\Sigma = \frac{\sin|h|/f}{|h|}\begin{pmatrix} h_a \\ \cot|h|/f \end{pmatrix},$$

$$|h| = \sqrt{h_a h_a},\ a = 1,2,3,4$$

With $|h|U \equiv \begin{pmatrix} h_4 + ih_3 & h_2 + ih_1 \\ -(h_2 - ih_1) & h_4 - ih_3 \end{pmatrix} = (\widetilde{\phi}, \phi)$ we find:

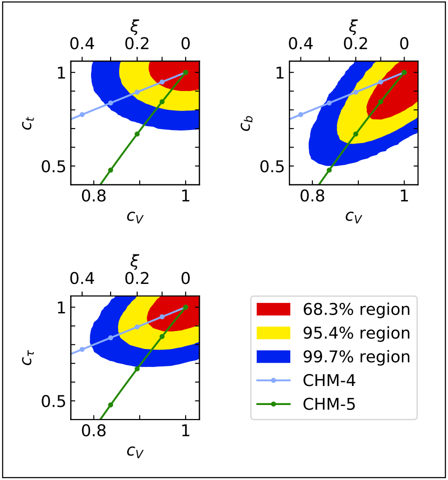
$$\mathcal{L}_{\text{kin}} = \frac{1}{2}\partial_\mu|h|\partial^\mu|h| + \frac{f^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle(\sin|h|/f)^2$$

The Minimal Composite Higgs Model

Agashe *et al.* [hep-ph/0412089,NPB], Contino *et al.* [hep-ph/0612048,PRD]

In the coset $SO(5)/SO(4)$: $\boxed{\xi = \frac{v^2}{f^2}}$

- $c_V = \sqrt{1-\xi}$
  universal
- $c_\psi^{(4)} = \sqrt{1-\xi}$ or $c_\psi^{(5)} = \frac{1-2\xi}{\sqrt{1-\xi}}$
  representation dependent
- **4** : $\xi < 0.22, \quad f > 530$ GeV
  5 : $\xi < 0.12, \quad f > 710$ GeV



de Blas/Eberhardt/CK [1803.00939,JHEP]

An Example, the SM Singlet Extension

$$\mathcal{L}_{\mathsf{SM+S}} \;=\; \mathcal{L}_{\mathsf{SM}} + \partial^\mu S \partial_\mu S + \frac{\mu_2^2}{2} S^2 - \frac{\lambda_2}{4} S^4 - \frac{\lambda_3}{2} \phi^\dagger \phi S^2$$

S: real scalar singlet with Z_2 symmetry

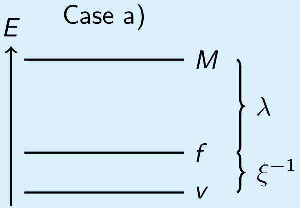
Schabinger/Wells [hep-ph/0509209], Patt/Wilczek [hep-ph/0605188], Robens/Stefaniak [1601.07880], Englert/Plehn/Zerwas/Zerwas [1106.3097], Buttazzo/Sala/Tesi [1505.05488], Freitas/López-Val/Plehn [1607.08251]

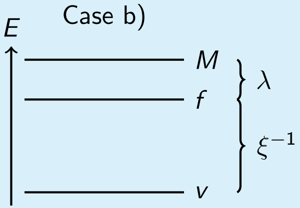
In physical parameters: $m, v, M, \sin\chi$, and $\xi = \frac{v^2}{f^2} = \frac{v^2}{v^2+v_s^2}$

$$\begin{aligned} V(h,H) =& \tfrac{1}{2} m^2 h^2 + \tfrac{1}{2} M^2 H^2 - d_1 h^3 - d_2 h^2 H - d_3 h H^2 - d_4 H^3 \\ & - z_1 h^4 - z_2 h^3 H - z_3 h^2 H^2 - z_4 h H^3 - z_5 H^4 \end{aligned}$$

$$d_i \;=\; d_i(m^2, M^2, v, \xi, \sin\chi), \qquad z_i \;=\; z_i(m^2, M^2, v, \xi, \sin\chi)$$

We distinguish 2 possible hierarchies.





$$|\lambda_i| \lesssim 32\pi^2,\quad \xi, \sin\chi = \mathcal{O}(1),\quad m \sim v \sim f \ll M$$

$$\lambda_i = \mathcal{O}(1),\quad \xi, \sin\chi \ll 1,\quad m \sim v \ll f \sim M$$

Buchalla/Catà/Celis/CK [1608.03564,NPB]

Integrate out H: solve equation of motion

$$H = H_0 + \frac{H_1}{M} + \frac{H_2}{M^2} + \dots$$

Case a), strong coupling, generates the $EWCh\mathcal{L}$.

$$H \quad = \quad H_0 \; + \; \frac{H_1}{M} \; + \; \frac{H_2}{M^2} \; + \; \dots$$

$$H_0 = H_0(h) = H_{0,2}\left(\frac{h}{v}\right)^2 + H_{0,3}\left(\frac{h}{v}\right)^3 + H_{0,4}\left(\frac{h}{v}\right)^4 + \dots$$

(closed-form solution to all orders in h)

$\rightarrow$ No $\frac{1}{M}$ suppression, but arbitrarily high canonical dimension

$\rightarrow$ Expansion in chiral dimensions $\rightarrow$ $EWCh\mathcal{L}$

LO:

$$\mathcal{L}_{\text{LO}} = \mathcal{L}_{\text{kin}} - V(h) + \mathcal{L}_{\text{Yuk}}(h) + \frac{v^2}{4}\langle (D_\mu U)(D^\mu U^\dagger)\rangle\,(1 + F_U(h))$$

NLO ($1/M^2$):

$\mathcal{O}_{D1}, \mathcal{O}_{D7}, \mathcal{O}_{D11}, \dots$ of Buchalla/Catà/CK [1307.5017,NPB]

Case b), weak coupling, generates the SMEFT.

$$H \quad = \quad H_0 \ + \ \frac{H_1}{M} \ + \ \frac{H_2}{M^2} \ + \ \dots$$

$$H_0 = 0, \qquad H_1 = -\frac{\lambda_3 v_H}{2M}\phi^\dagger\phi$$

$\rightarrow$ Always $\frac{1}{M}$ suppression

$\rightarrow$ Expansion in canonical dimensions $\rightarrow$ SMEFT

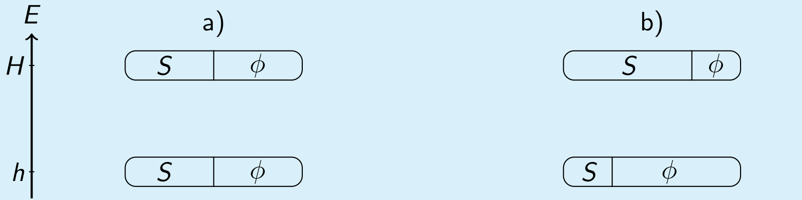
LO:

SM with renormalized couplings

NLO $(1/M^2)$:

$$\mathcal{L}_{\text{NLO}} = \frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M^2}\partial^\mu(\phi^\dagger\phi)\partial_\mu(\phi^\dagger\phi)$$

The physical picture helps to relate the two EFTs.



a) strongly coupled, large mixing

$\rightarrow$ EFT in terms of U, h

b) weakly coupled, small mixing

$\rightarrow$ EFT in terms of ϕ

In the limit $\sin\chi \ll 1$, we should recover the decoupling scenario.

$\rightarrow$ Indeed, $\sin\chi \ll 1$ corresponds to $\xi \ll 1$ and we do recover the decoupling case.

Renormalization of SMEFT, an explicit example

Starting from

$$\mathcal{Q}_\phi = \left(\phi^\dagger\phi\right)^3 = \frac{1}{8}\left(\varphi_i\varphi_i\right)^3,$$

we find

$$a_{ij}^{\mu\nu} = 0, \qquad b_{ij}^{\mu} = 0, \qquad c_{ij} = -\frac{3}{4}(\hat\varphi_a\hat\varphi_a)^2\delta_{ij} - 3(\hat\varphi_a\hat\varphi_a)\hat\varphi_i\hat\varphi_j,$$

$$Y_{ij} = \left(\left(\frac{\lambda}{2} + \frac{g^2}{4}\right)\hat\varphi_a\hat\varphi_a - m^2\right)\delta_{ij} + \left(\lambda - \frac{g^2}{4}\right)\hat\varphi_i\hat\varphi_j - g'^2(t_R^3\hat\varphi)_i(t_R^3\hat\varphi)_j.$$

Therefore

$$\mathrm{tr}\, cY = -\left(54\lambda + \frac{9}{2}g^2 + \frac{3}{2}g'^2\right)\left(\phi^\dagger\phi\right)^3 + 24m^2\left(\phi^\dagger\phi\right)^2.$$

gives with $K_{(\phi)} = 6(3g^2 + g'^2 - \gamma_\phi)$

$$\beta_\phi \supseteq \left(54\lambda - \frac{27}{2}g^2 - \frac{9}{2}g'^2 + 6\gamma_\phi\right)C_\phi, \qquad \text{and} \qquad \beta_\lambda \supseteq 48\frac{m^2}{\Lambda^2}C_\phi.$$